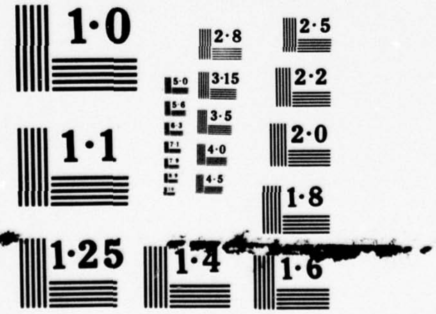
AD-A068 222 CALIFORNIA UNIV BERKELEY GEOTECHNICAL ENGINEERING F/G 8/13
EVALUATION OF THREE CONSTITUTIVE MODELS FOR SOILS.(U)
MAR 79 P C LUCIA, J M DUNCAN DACW39-78-M-2077
UNCLASSIFIED UCB/GT/78-05 WES-MPGL-79-3 NL

1 OF 2
ADA
068222

068222



NATIONAL BUREAU OF STANDARDS
MICROCOPY RESOLUTION TEST CHART

LEVEL IV

MISCELLANEOUS PAPER GL-79-3

EVALUATION OF THREE CONSTITUTIVE MODELS FOR SOILS

by

Patrick C. Lucie, James M. Duncan

College of Engineering
University of California
Berkeley, California 94720

March 1979

Final Report

Approved For Public Release; Distribution Unlimited

Prepared for Office, Chief of Engineers, U. S. Army
Washington, D. C. 20314
(Work Unit CWIS 31173, Task 25c)

Under Contract No. DACW39-78-M-2077

Monitored by Geotechnical Laboratory
U. S. Army Engineer Waterways Experiment Station
P. O. Box 631, Vicksburg, Miss. 39180

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
Miscellaneous Paper GL-79-3

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

9

4. TITLE (and Subtitle)
6 EVALUATION OF THREE CONSTITUTIVE MODELS FOR SOILS.

5. TYPE OF REPORT & PERIOD COVERED
Final report

6. PERFORMING ORG. REPORT NUMBER
Report No. UCB/GT/78-05

7. AUTHOR(s)
10 Patrick C. Lucia
James M. Duncan

8. CONTRACT OR GRANT NUMBER(s)
14
15 DACW39-78-M-2077

9. PERFORMING ORGANIZATION NAME AND ADDRESS
College of Engineering
Department of Civil Engineering
University of California, Berkeley, Calif. 94720

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
Work Unit CWIS 31173
Task 25c

11. CONTROLLING OFFICE NAME AND ADDRESS
Office, Chief of Engineers, U. S. Army
Washington, D. C. 20314

12. REPORT DATE
11 Mar[redacted]79

13. NUMBER OF PAGES
92
12 100p.

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
U. S. Army Engineer Waterways Experiment Station
Geotechnical Laboratory
P. O. Box 631, Vicksburg, Miss. 39180

15. SECURITY CLASS. (of this report)
Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)
Approved for public release; distribution unlimited.

18 WES 19 MP-GL-79-3

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
Constitutive models
Dilatancy (soils)
Finite element method
Mathematical models
Soil deformation
Stress-strain relations (soils)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The finite element method provides a powerful technique for analysis of stresses and movements in earth masses, and it has already been applied to a number of practical problems including embankment dams, open excavations, braced excavations, and a variety of soil-structure interaction problems.

If the results of soil deformation analyses are to be realistic and meaningful, it is important that the stress-strain characteristics of the soil be

cont'

(Continued)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

JOB

411056

79 05 01 005

20. ABSTRACT (Continued).

Con't

represented in the analyses in a reasonable way. This is difficult because the stress-strain characteristics of soils are extremely complex, and the behavior of soil is nonlinear, inelastic, and highly dependent on the magnitudes of the stresses in the soil.

The purpose of this report is to compare the characteristics of the hyperbolic, the Cam Clay and the Al-Shawaf and Powell stress-strain models for use in finite element analyses of earth masses. The principal objective of the study was to investigate the potential advantages and disadvantages of the Al-Shawaf - Powell model as compared to the other, more widely used stress-strain relationships.

The hyperbolic stress-strain relationships were developed in an attempt to provide a simple framework encompassing the most important characteristics of soil stress-strain behavior, using the data available from conventional laboratory tests. These relationships have been used in finite element analyses of a number of different types of static soil mechanics problems and values of the hyperbolic parameters have now been determined for about 150 different soils. Duncan et al. (1978) outlined procedures for determination of the hyperbolic stress-strain and volume change parameters for use in nonlinear finite element analyses of stresses and movements in earth masses.

The Cam Clay model was developed at Cambridge, and has recently been modified by Chang and Duncan (1978) for use in finite element analyses of stresses and movements in earth masses constructed of compacted materials, such as earth dams. Chang and Duncan outlined the procedures for determination of stress-strain and volume change parameters to provide the best fit between calculated and measured triaxial data.

A model recently developed by Al-Shawaf and Powell differs from the hyperbolic and Cam Clay model in two important respects.

(1) It models shear dilatancy.

(2) The tangent moduli are obtained by numerical procedures rather than through the use of analytical expressions.

This report is concerned with evaluating the following characteristics of the Al-Shawaf - Powell model:

1. The ability of the model to reproduce triaxial test data for soils exhibiting dilatant behavior.

2. The procedures required to develop the stress-strain and volume change parameters.

3. The potential for use of the model in practical geotechnical problems.

In the following sections the characteristics of the three models are described, the procedures for evaluating the parameters are explained, and comparisons between calculated and measured stress-strain curves for three dilatant soils are presented.

THE CONTENTS OF THIS REPORT ARE NOT TO BE USED FOR ADVERTISING, PUBLICATION, OR PROMOTIONAL PURPOSES. CITATION OF TRADE NAMES DOES NOT CONSTITUTE AN OFFICIAL ENDORSEMENT OR APPROVAL OF THE USE OF SUCH COMMERCIAL PRODUCTS.

PREFACE

This report was prepared by Messrs. P. C. Lucia and J. M. Duncan under Contract No. DACW39-78-M-2077 between the U. S. Army Engineer Waterways Experiment Station (WES) and the University of California at Berkeley. The research was sponsored by the Office, Chief of Engineers, U. S. Army, under the Civil Works Program CWIS No. 31173, "Special Studies for Civil Works Problems," Task 25c, concerning potential improvements to the computer code CON2D for analysis of thin core earth and rock-fill dams. The objective was to explore the practical application of the Al-Shawaf-Powell model, which models shear dilatant soil behavior.

The contract was monitored by Mr. C. L. McAnear, Chief, Soil Mechanics Division, Geotechnical Laboratory (GL), under the general supervision of Mr. J. P. Sale, Chief, GL. The Contracting Officer was COL J. L. Cannon, CE, Commander and Director of the WES. Mr. F. R. Brown was Technical Director.

CONTENTS

INTRODUCTION

The finite element method provides a powerful technique for analysis of stresses and movements in earth masses, and it has already been applied to a number of practical problems including embankment dams, open excavations, braced excavations, and a variety of soil-structure interaction problems.

If the results of soil deformation analyses are to be realistic and meaningful, it is important that the stress-strain characteristics of the soil be represented in the analyses in a reasonable way. This is difficult because the stress-strain characteristics of soils are extremely complex, and the behavior of soil is nonlinear, inelastic, and highly dependent on the magnitudes of the stresses in the soil.

The purpose of this report is to compare the characteristics of the hyperbolic, the Cam Clay and the Al-Shawaf and Powell stress-strain models for use in finite element analyses of earth masses. The principal objective of the study was to investigate the potential advantages and disadvantages of the Al-Shawaf - Powell model as compared to the other, more widely used stress-strain relationships.

The hyperbolic stress-strain relationships were developed in an attempt to provide a simple framework encompassing the most important characteristics of soil stress-strain behavior, using the data available from conventional laboratory tests. These relationships have been used in finite element analyses of a number of different types of static soil mechanics problems and values of the hyperbolic parameters have now been determined for about 150 different soils. Duncan et al. (1978) outlined procedures for determination of the hyperbolic stress-strain and volume change parameters for use in nonlinear finite element analyses of stresses and movements in earth masses.

The Cam Clay model was developed at Cambridge, and has recently been modified by Chang and Duncan (1978) for use in finite element analyses of stresses and movements in earth masses constructed of compacted materials, such as earth dams. Chang and Duncan outlined the

procedures for determination of stress-strain and volume change parameters to provide the best fit between calculated and measured triaxial data.

A model recently developed by Al-Shawaf and Powell differs from the hyperbolic and Cam Clay model in two important respects.

1) It models shear dilatancy.

2) The tangent moduli are obtained by numerical procedures rather than through the use of analytical expressions.

This report is concerned with evaluating the following characteristics of the Al-Shawaf - Powell model:

1. The ability of the model to reproduce triaxial test data for soils exhibiting dilatant behavior.

2. The procedures required to develop the stress-strain and volume change parameters.

3. The potential for use of the model in practical geotechnical problems.

In the following sections the characteristics of the three models are described, the procedures for evaluating the parameters are explained, and comparisons between calculated and measured stress-strain curves for three dilatant soils are presented.

AL-SHAWAF - POWELL MODEL

The Al-Shawaf - Powell model was developed for nonlinear analysis of soil deformations, and was described in the paper in Appendix A. This procedure uses a numerical modelling process in which the soil is modelled as being linear during each increment of loading. By varying the values of the parameters when the stresses reach a new "zone", the nonlinear behavior of soil is modeled. The procedure differs from most other models in that the tangent moduli are obtained by numerical procedures rather than analytical expressions. Its greatest attraction is the fact that it can model shear dilatancy in soils.

The purpose of this report is to present the results of a study which include

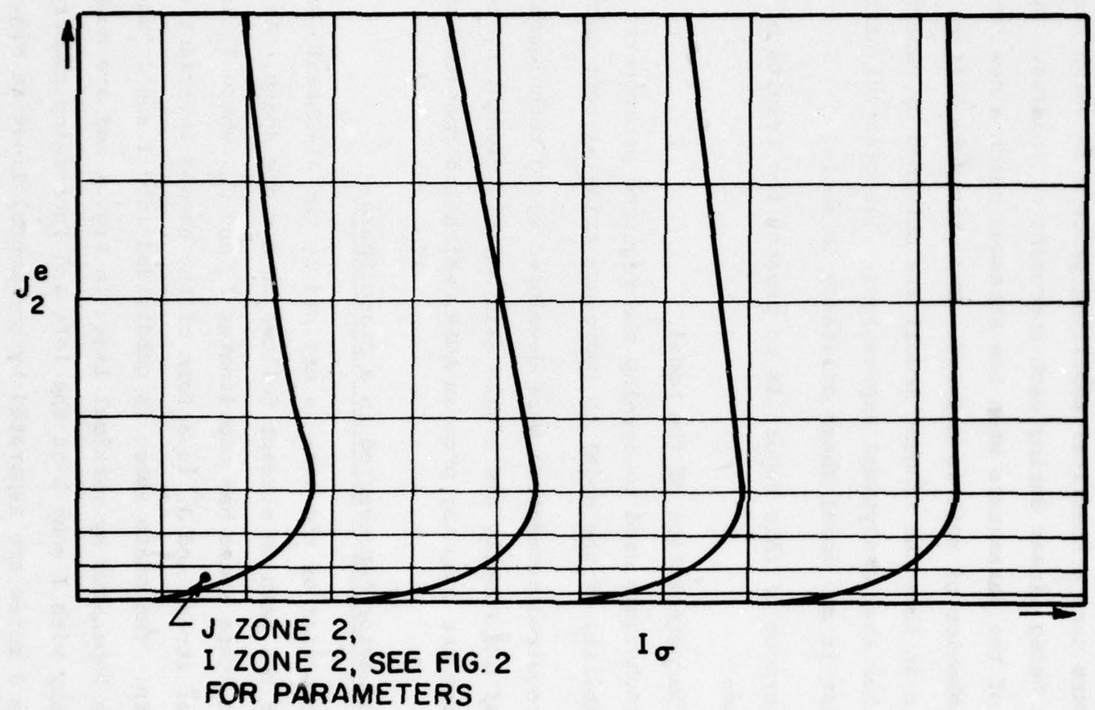
1. Characteristics of the model
2. Techniques used to develop the required parameters
3. Ability of the model to reproduce triaxial test data

The required parameters were developed using techniques presented by Al-Shawaf and Powell, and single axisymmetric element systems were studied using the computer program ANSR, which incorporates the model.

Nonlinear Behavior Represented in a State Space

The behavior of the soil is defined by the Al-Shawaf - Powell model in terms of the path an element follows in a state space. As shown in Fig. 1, the state space has coordinates I_σ and J_2^e, where I_σ is the first invariant of stress and J_2^e is a form of the second invariant of deviatoric strain. The state space is subdivided into I and J "zones". The I zones are separated by vertical lines in Fig. 1 and are numbered consecutively with I zone 1 at the left and increasing numbers to the right. The J zones are separated by horizontal lines in Fig. 1 and are numbered consecutively with J zone 1 at the bottom and increasing numbers upward.

The optimum width of the J zones is determined by plotting triaxial test data in the state space and spacing the J zones so that the slopes of the curves vary consistently from one J zone to the next.



FOR TRIAXIAL DATA:

$$J_2^e = \frac{\sqrt{3}}{2}\ (\epsilon_a - \epsilon_v/3)$$

$$I_\sigma = \sigma_1 + 2\sigma_3$$

Fig. 1 STATE SPACE EMPLOYED IN AL-SHAWAF - POWELL MODEL

Typically, more J zones are required in the initial portion of these curves, which correspond to the initial portion of the triaxial stress-strain curves. At strains past the peak, the zones can become larger because the variation in slope with strain is much smaller in this region.

The optimum width of the I zones is determined by observing the variation of the bulk shear modulus, K, shear modulus, G, and dilatancy ratio, D with increasing stress. The I zones are chosen so that all 3 parameters vary consistently from one I zone to the next. More I zones are usually required in the initial portions of the curves where the variation with increasing stress is the greatest. At higher stresses the variations become smaller and larger I zones can usually be used.

The parameters required for the Al-Shawaf - Powell model are shown on Fig. 2 and listed in Table 1. The bulk modulus, K, and the bulk modulus factor, f, are derived from isotropic triaxial consolidation tests. The value of K is calculated from the volume change behavior of the soil during loading. The value of f is the ratio of the bulk modulus during unloading to the bulk modulus during loading, and is assumed to be constant over the stress range of interest. The modulus during unloading is higher than the modulus during loading, so that the value of f will always be greater than 1. In general, the value of the bulk modulus will increase initially and remain relatively constant as the tendency for volume change decreases at higher values of isotropic stress. Each I zone is assigned the midpoint value of K and all J zones within a given I zone are assigned the same value of K. All J and I zones in the state space are assigned the same value of f.

The tangent values of shear modulus, G, are proportional to the slopes of the path in the state space, (Fig. 1). Within a given J zone the variation of G with stress is plotted as shown in Fig. 2. There are as many curves showing the variation of G with I_σ as there are J zones. The values of G are also proportional to the slopes of the triaxial stress-strain curves. It can be seen that values of G for the first J zones correspond to the initial slopes of the stress-strain

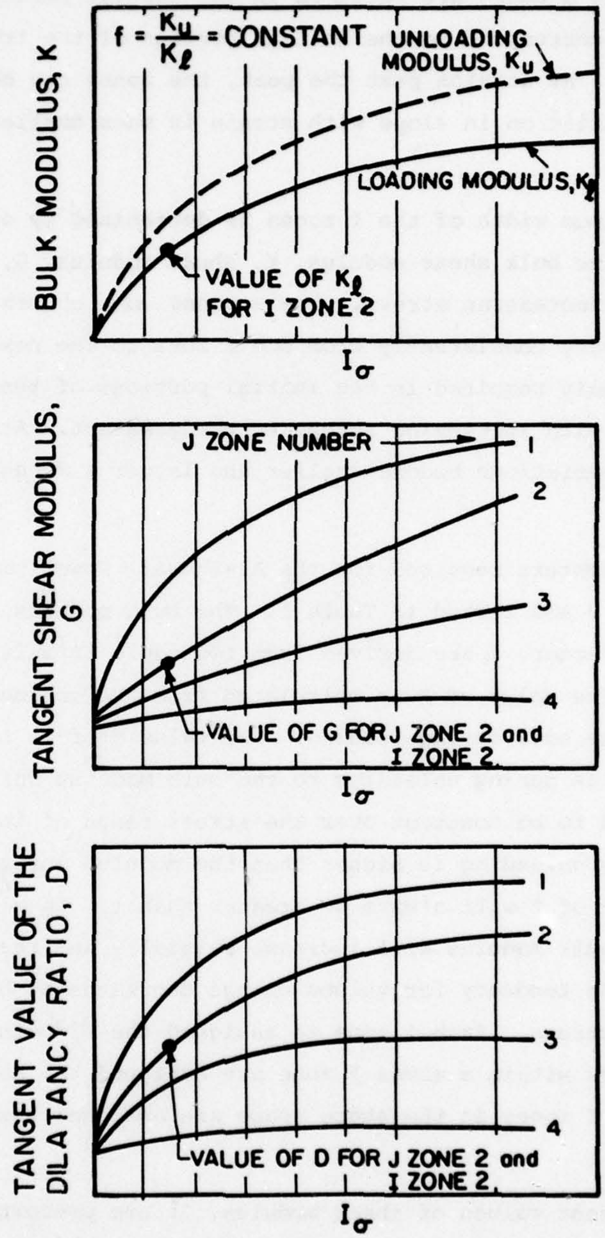


Fig. 2 DETERMINATION OF THE PARAMETERS K, G AND D FOR A GIVEN J AND I ZONE

Table 1. Summary of Parameters for Al-Shawaf - Powell Model

Parameter	Name	Function
K	Tangent bulk modulus	Relates volumetric strain to mean stress
G	Tangent shear modulus	Relates deviator stress to strain
D	Tangent value of dilatancy ratio	Relates mean strain to deviatoric strain
f	Bulk modulus factor	Relates unloading bulk modulus to loading bulk modulus
*	Triaxial extension ratio	Relates strain in triaxial compression to strain in triaxial extension

*No symbol has been assigned to this parameter by Al-Shawaf and Powell.

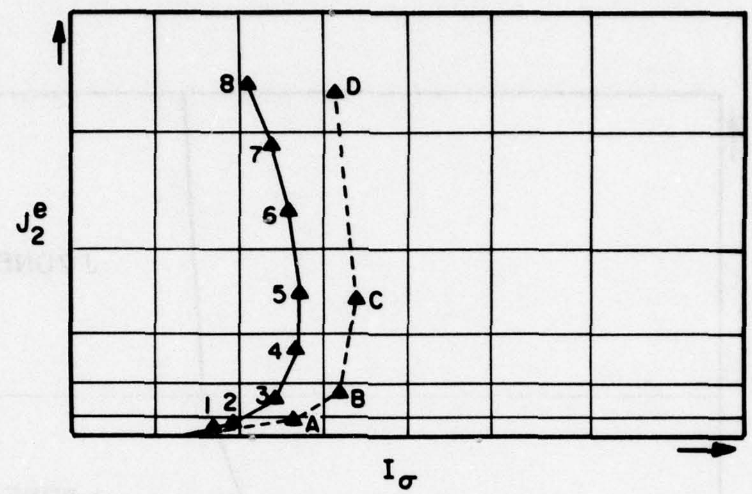
curves, and are relatively large. The values of G for higher-numbered J zones are smaller, reflecting the nonlinear behavior of soil. For dilatant material, the value of G may become negative for combinations of I and J which correspond to conditions involving strains past the peak. In assigning values of G to zones in the state space, the mid-point value of G within the I zone is chosen. For example, the point labelled in Fig. 2 corresponds to I zone 2 and J zone 2 in Fig. 1.

The tangent value of the dilatancy ratio, D, is primarily related to the shape of the volume change curve. The value of D is plotted versus stress for each J zone. The tendency for compressive volume changes decreases with increasing strain, and that tendency is reflected by the fact that the values of D decrease with increasing J zone number. Negative values of D reflect dilatant behavior in the soil. The values of D are assigned to zones in the same way as for G. Typical variations of G and D with I and J are shown in Fig. 2.

As mentioned previously, it is important that sizes of the zones are chosen so that the values of the parameters vary consistently from one zone to the next. It is equally important that the increments of load analyzed should follow paths in the state space which proceed from zone to zone without skipping a significant number of zones. Fig. 3 shows the significance of the number of load increments on the calculated stress-strain behavior. Initial load increments should be small enough so that several increments fall within the first three or four J zones. The example illustrated in Fig. 3 shows that the stress-strain curve is crudely approximated when a small number of increments is used, as shown by the curves labelled A-B-C-D. More increments model the data more closely, as shown by the curves 1-2-3-4-5-6-7-8.

Variation of the Tangent Shear Modulus, G, in State Space

The values of the tangent shear modulus, G, are proportional to the slope of the path in the state space and are also proportional to the slope of the triaxial stress-strain curve. The value of G for a given J zone is calculated using the following equation, as shown in Fig. 4:



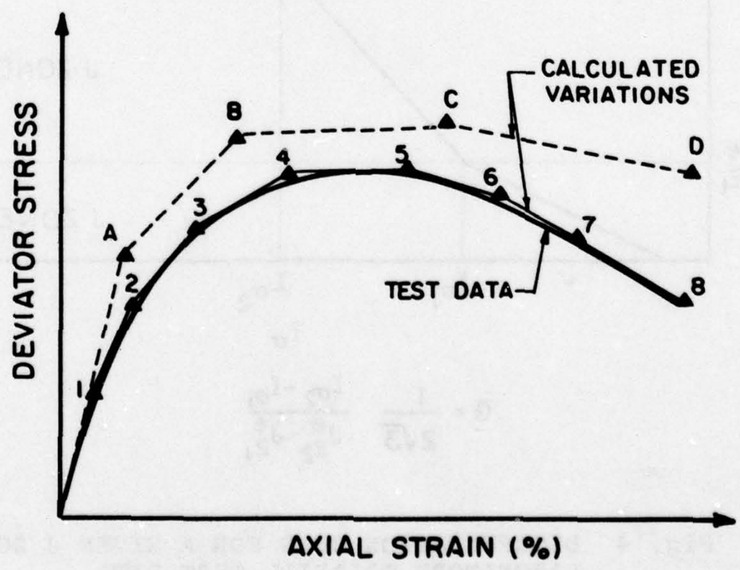
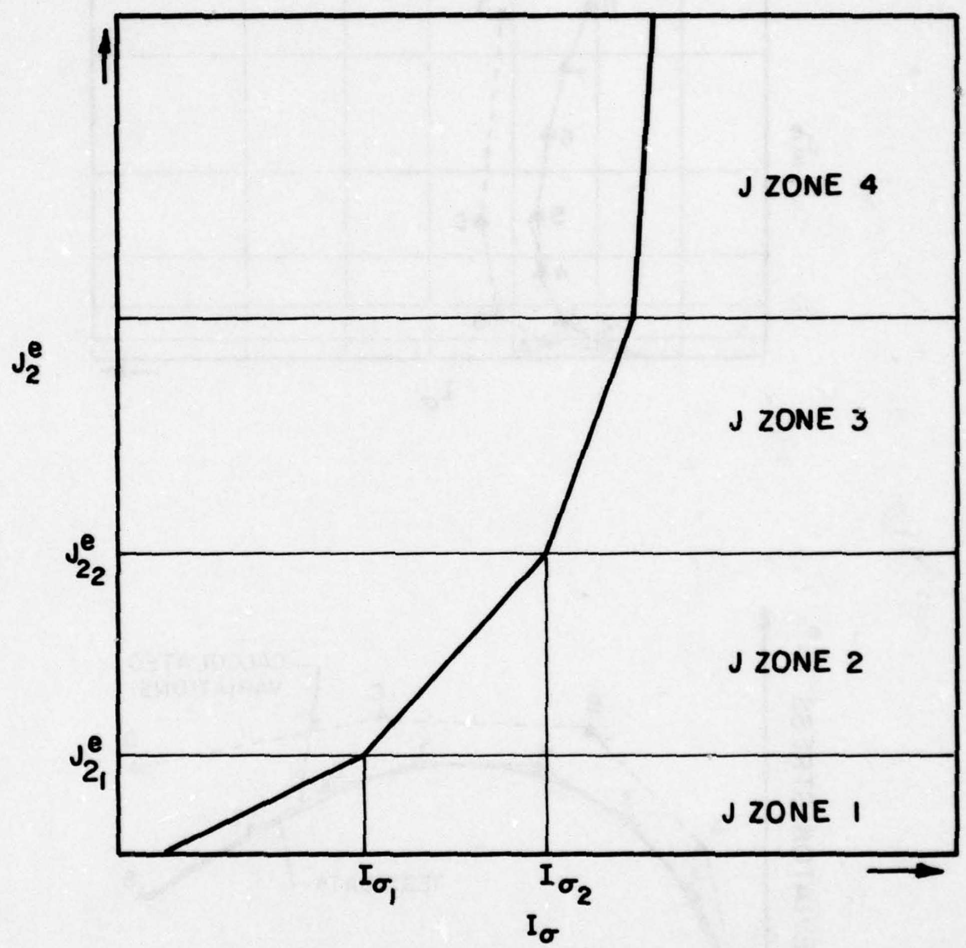


Fig. 3 SIGNIFICANCE OF PATH IN STATE SPACE



$$G = \frac{1}{2\sqrt{3}} \quad \frac{I_{\sigma_2} - I_{\sigma_1}}{J_{2_2}^e - J_{2_1}^e}$$

Fig. 4 DETERMINATION OF G FOR A GIVEN J ZONE FROM LABORATORY TRIAXIAL TEST DATA

$$G = \frac{1}{2\sqrt{3}} \; \frac{I_{\sigma_2} - I_{\sigma_1}}{J^e_{2_2} - J^e_{2_1}} \tag{1}$$

The values of I_σ are determined at the points where the path crosses the J zone boundaries. A value of G is calculated for each path within each J zone, and these values are plotted against the midpoint values of I_σ. Smooth curves are then drawn through the points as shown in Fig. 5. The variation of G with I_σ for each J zone is shown by these curves.

The magnitude of G affects only the slope of the stress-strain curves, and has no effect on the volume change behavior. The effect of G on stress-strain behavior is illustrated in Fig. 6. The initial loading portion of the stress-strain curve corresponds to positive values of G, the peak of the stress-strain curve results in a value of G equal to zero, and negative values of G are associated with a reduction in deviator stress beyond the peak. The shape of the stress-strain curve can therefore be adjusted by changes in the values of G. Increasing values of the shear modulus will result in a higher stress for a given strain as shown on Fig. 6, and will thus move the stress-strain curve to the left.

Variation of the Tangent Bulk Modulus, K, in State Space

The relationship between bulk modulus, K, and stress is derived from isotropic triaxial consolidation test results. The results can be plotted, as shown in Fig. 7, where the mean stress is equal to the consolidation pressure and the mean strain is equal to one-third the volumetric strain. The bulk modulus is calculated by determining the variation of the slope of the loading curve (the Bulk Modulus) with stress. The bulk modulus is then plotted versus I_σ as shown on Fig. 8.

The unloading modulus is calculated from the slope of the unloading portion of the curve. The unloading modulus will also vary with stress, but its variation is usually difficult to calculate because of the small strains involved. In the Al-Shawaf - Powell model the

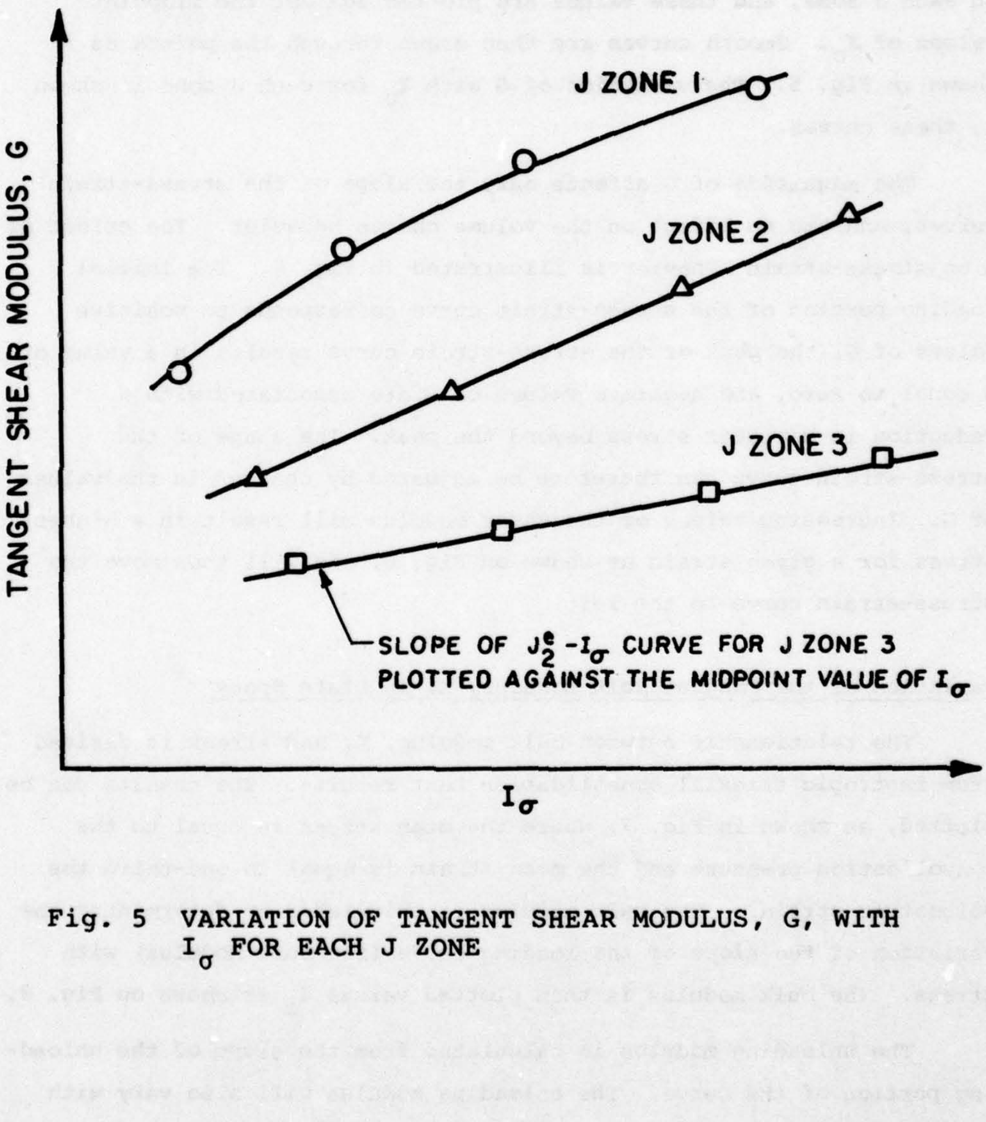


Fig. 5 VARIATION OF TANGENT SHEAR MODULUS, G, WITH I_σ FOR EACH J ZONE

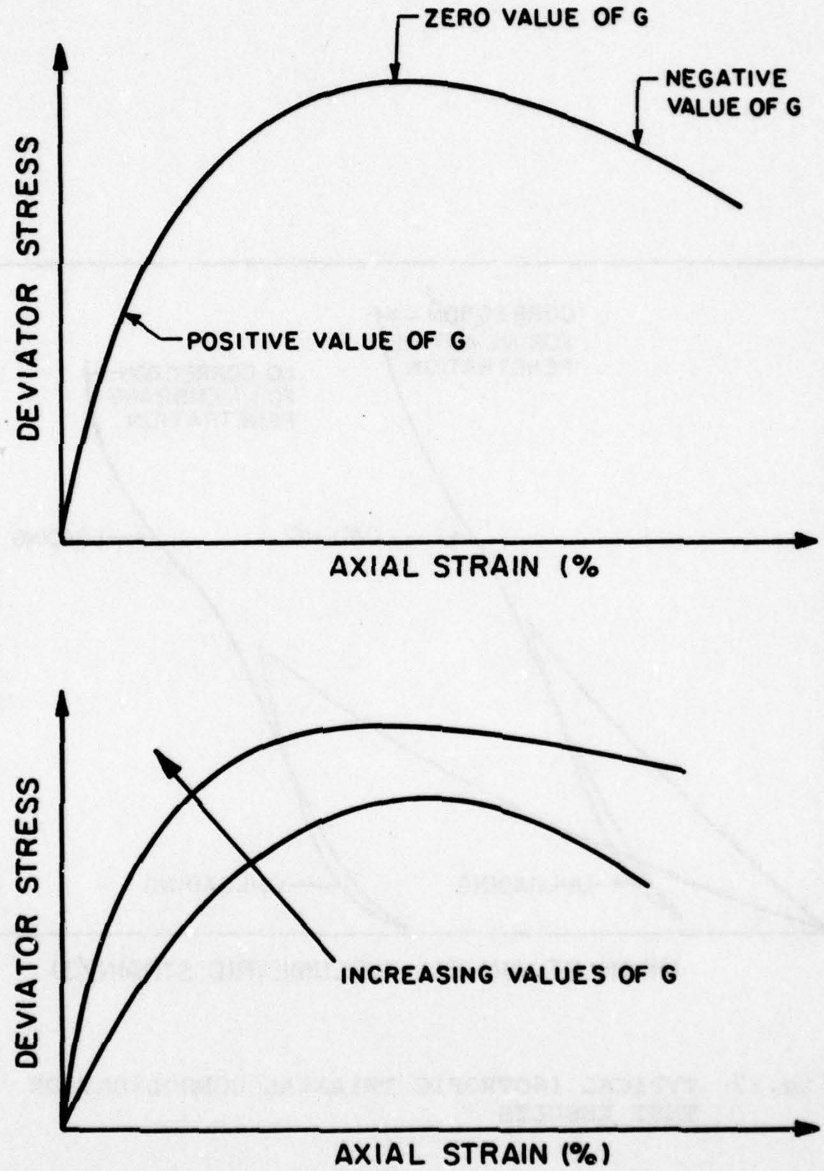


Fig. 6 EFFECT OF THE TANGENT SHEAR MODULUS, G, ON STRESS-STRAIN BEHAVIOR

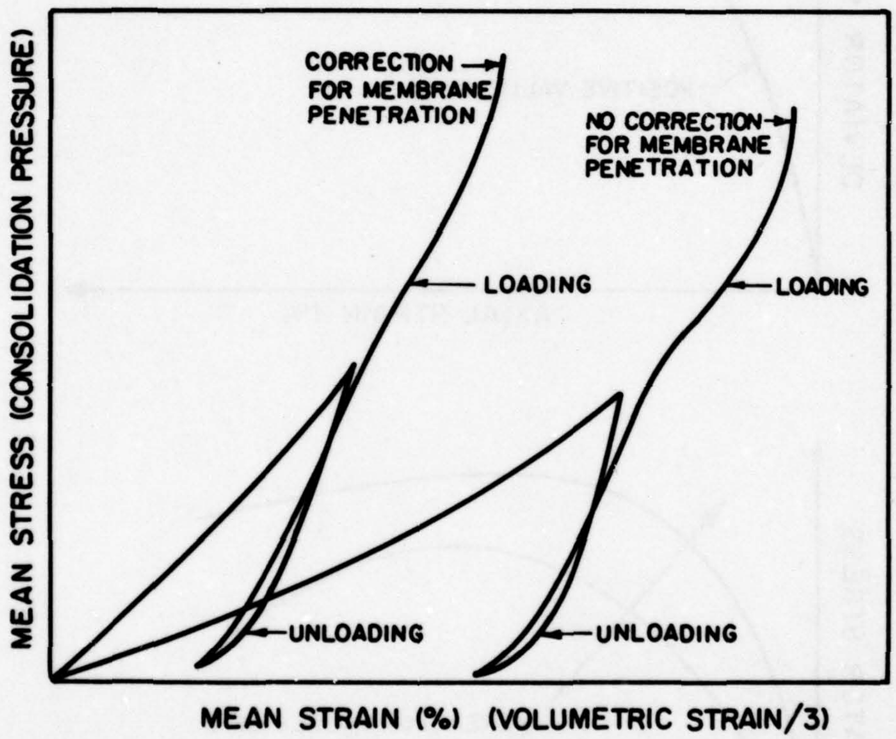


Fig. 7 TYPICAL ISOTROPIC TRIAXIAL CONSOLIDATION TEST RESULTS

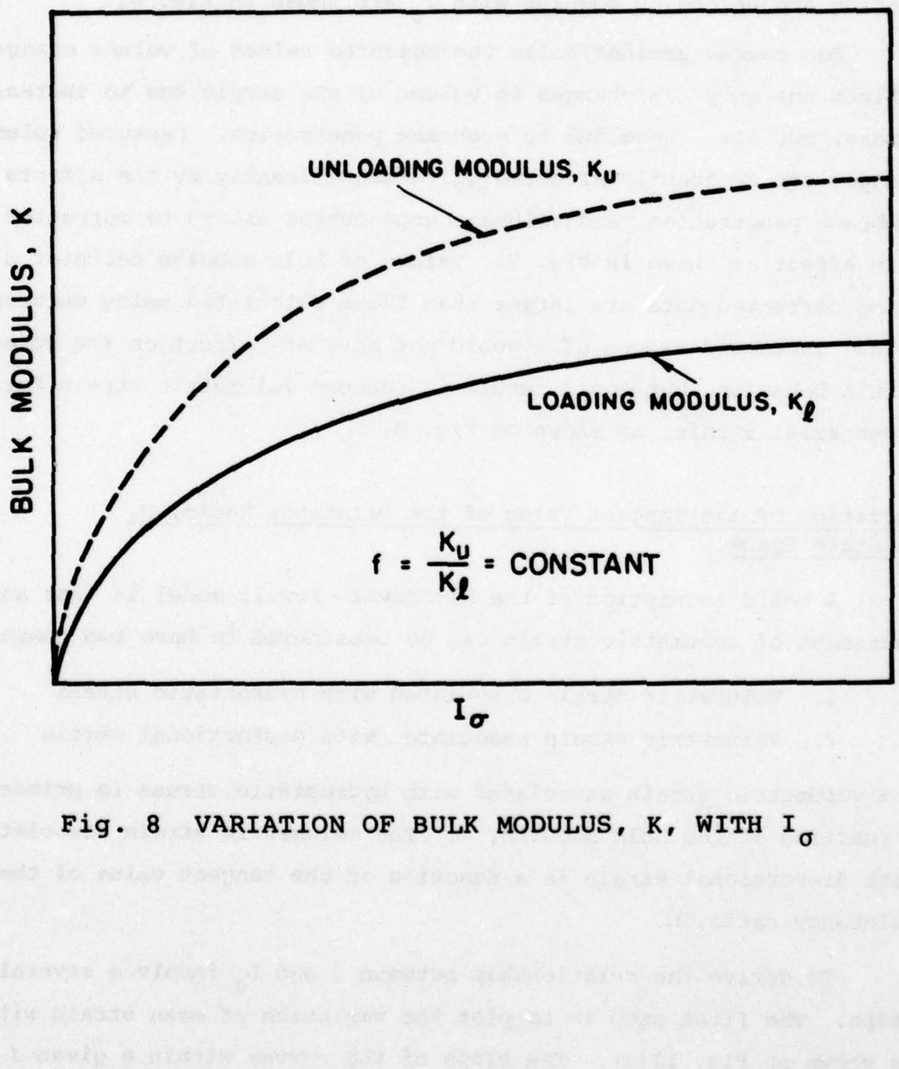


Fig. 8 VARIATION OF BULK MODULUS, K, WITH I_σ

assumption is made that the ratio between the loading modulus and unloading is a constant. One or two values of unloading modulus are calculated and corresponding values of the parameter f are determined, where f is the ratio of unloading to loading bulk modulus. Whenever unloading occurs during an analysis, the bulk modulus is multiplied by f to determine the value of unloading modulus. Typical variations of loading and unloading modulus with I_σ are shown in Fig. 8.

For coarse-grained soils the measured values of volume change reflect not only the changes in volume of the sample due to increased stress, but also those due to membrane penetration. Measured volume changes are frequently affected quite significantly by the effects of membrane penetration, and volume change curves should be corrected for this effect as shown in Fig. 7. Values of bulk modulus calculated using corrected data are larger than those calculated using uncorrected data. Increased values of K would not have any effect on the stress-strain behavior, but would result in reduced volumetric strain for a given axial strain, as shown on Fig. 9.

<u>Variation of the Tangent Value of the Dilatancy Ratio, D, in State Space</u>

A basic assumption of the Al-Shawaf - Powell model is that any increment of volumetric strain can be considered to have two components.

1. Volumetric strain associated with hydrostatic stress
2. Volumetric strain associated with distortional strain

The volumetric strain associated with hydrostatic stress is primarily a function of the bulk modulus, K. The volumetric strain associated with distortional strain is a function of the tangent value of the dilatancy ratio, D.

To derive the relationship between D and I_σ involves several steps. The first step is to plot the variation of mean strain with J as shown on Fig. 10(a). The slope of the curves within a given J zone are then calculated, and the values plotted against the corresponding values of I_σ, as shown on Fig. 10(b), with one curve for each J zone. The value of D can then be calculated from the variation of $d\varepsilon_m/dJ_2^e$

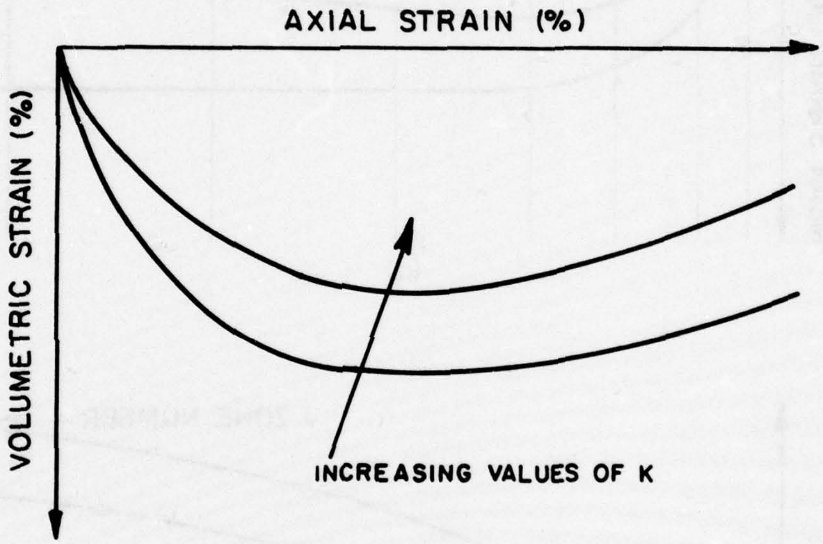


Fig. 9 EFFECT OF THE BULK MODULUS, K, ON VOLUME CHANGE BEHAVIOR

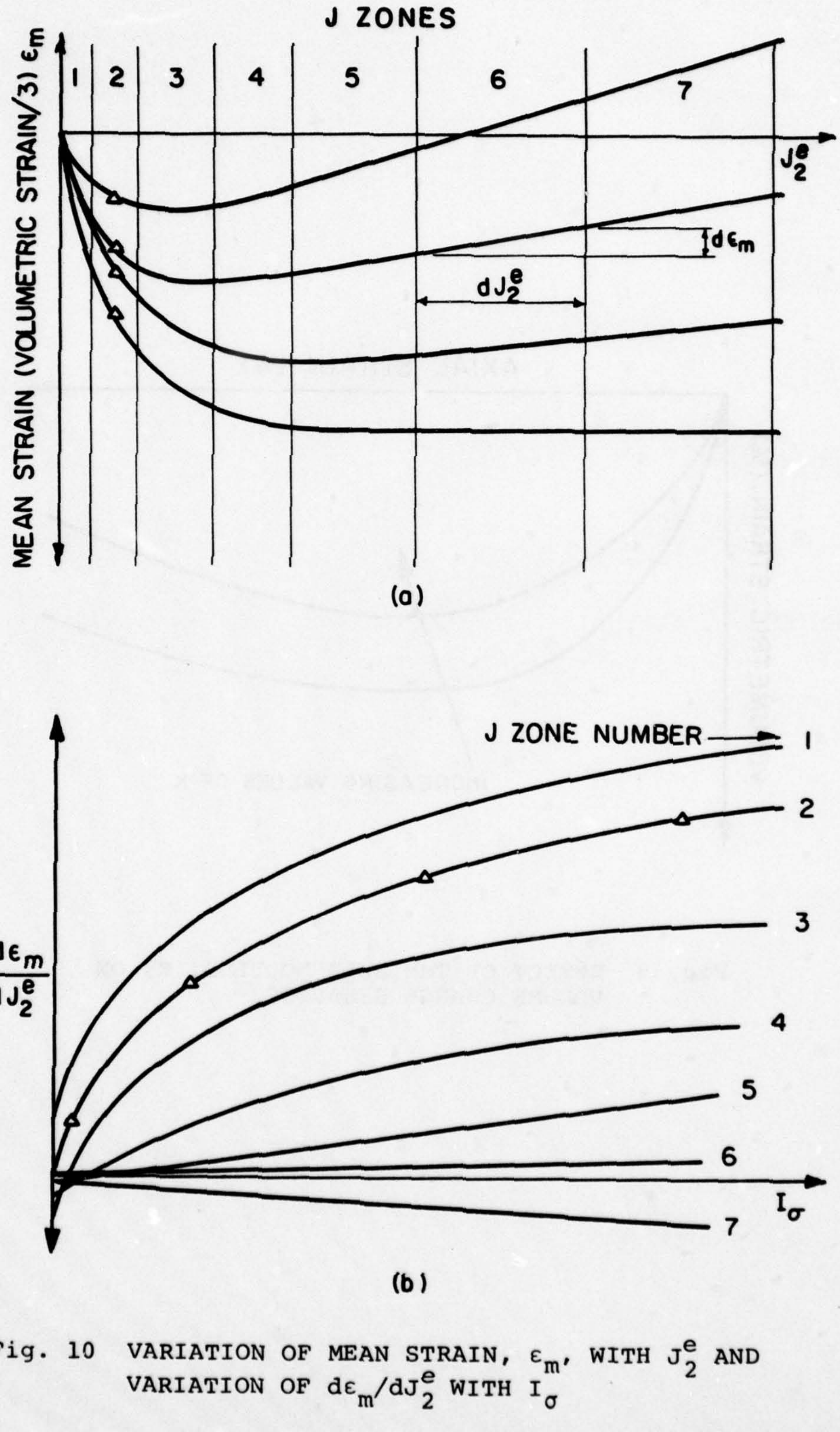


Fig. 10 VARIATION OF MEAN STRAIN, ε_m, WITH J_2^e AND VARIATION OF $d\varepsilon_m/dJ_2^e$ WITH I_σ

with I_σ, (Fig. 10(b)) the variation of G with I_σ (Fig. 5), and the variation of K with I_σ, (Fig. 8). For a given value of I_σ and a given J zone, D can be calculated from the following equation:

$$D = \frac{d\varepsilon_m}{dJ_2^e} - \frac{2G}{\sqrt{3}\,K} \tag{2}$$

If the given value of I_σ corresponds to an unloading condition (reduced values of I_σ), the value of K in equation (2) is multiplied by the bulk modulus factor, f. The variation of D with I_σ is shown on Fig. 11.

Variations in the value of D have no effect on the stress-strain curves, but will affect the volume change curve as shown on Fig. 12. Positive values of D indicate that the volume behavior is compressive, while negative values of D indicate dilative behavior.

Techniques for Determining Values of the Al-Shawaf - Powell Parameters from Laboratory Test Results

The values of the Al-Shawaf - Powell parameters can be determined using data from triaxial tests. The detailed procedures for evaluating the parameters are described in the following paragraphs, and are illustrated using data for a sandy gravel from Costa Rica.

Selecting Data and Eliminating Inconsistencies

The first step in evaluating the parameters is to select data appropriate to the problem being analyzed. In the case of natural soils, the laboratory tests must be performed using undisturbed specimens. In the case of fill materials, the laboratory tests must be performed using specimens compacted to the same density and water content as in the field. In both cases the drainage conditions in the laboratory tests should correspond to those in the problem being analyzed.

Tests should be performed to bracket the range of pressures of interest in the analysis. The parameters in the state space are determined by calculating the parameters for the path followed by the triaxial test and interpolating the data between those points. The test

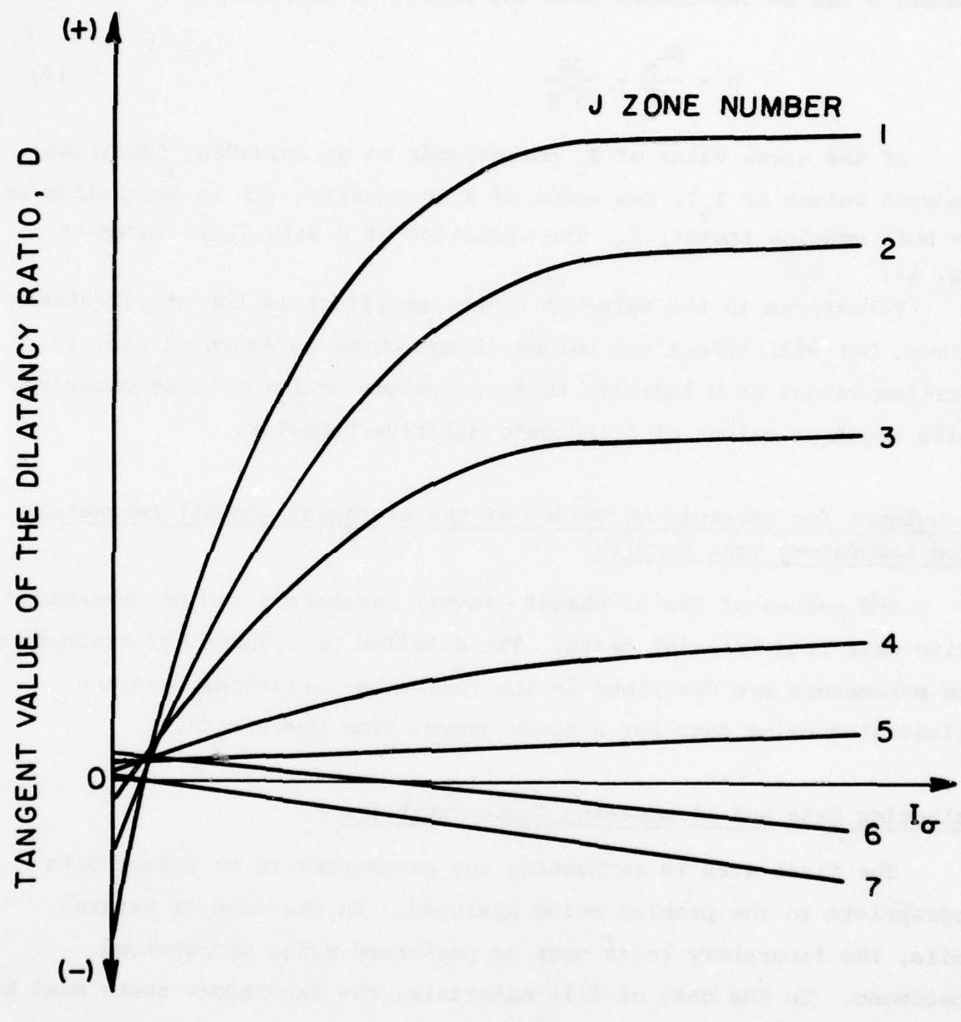


Fig. 11 VARIATION OF THE TANGENT VALUE OF THE DILATANCY VALUE, D, WITH I_σ

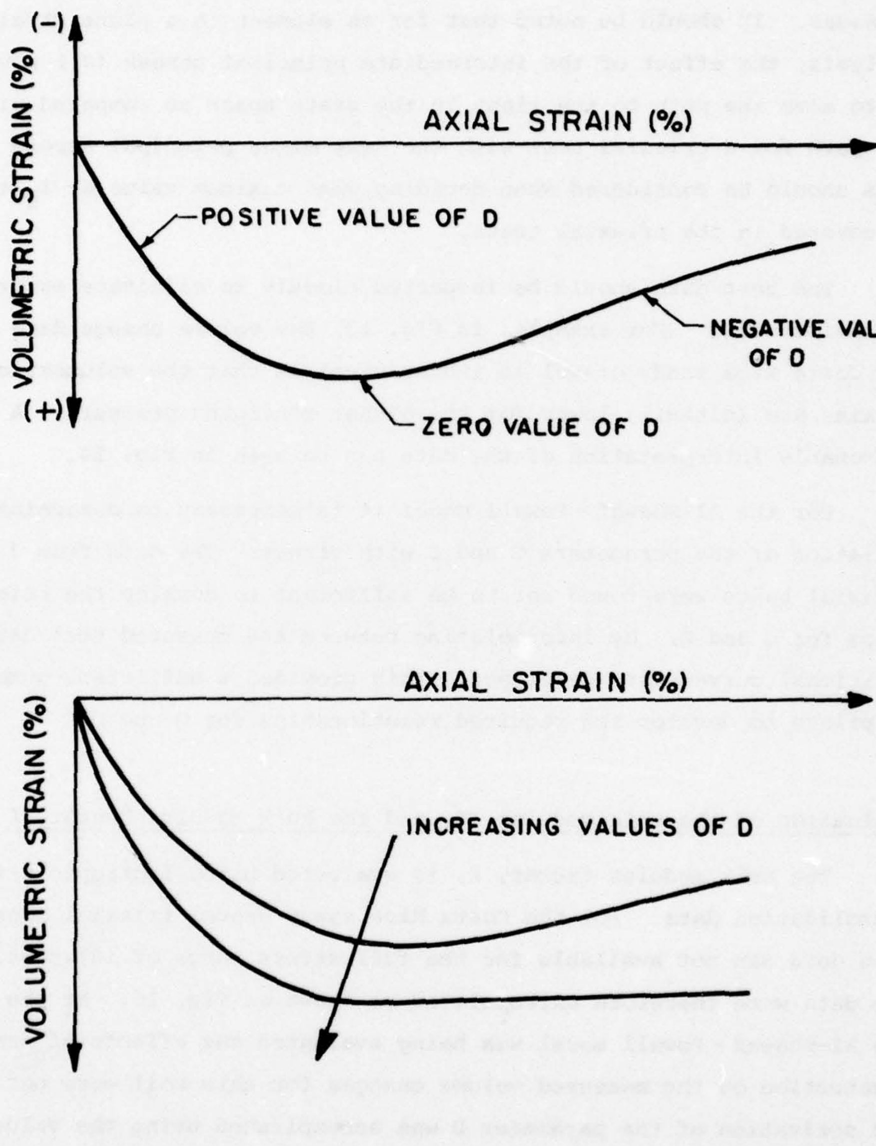


Fig. 12 EFFECT OF THE VARIATION OF THE TANGENT VALUE OF THE DILATANCY RATIO ON VOLUME CHANGE BEHAVIOR

data should provide data slightly beyond the maximum anticipated stresses. It should be noted that for an element in a plane strain analysis, the effect of the intermediate principal stress (σ_2) would be to move the path to the right in the state space as compared with the path for a triaxial test with the same minor principal stress (σ_3). This should be considered when deciding what maximum value of I_σ should be covered in the triaxial tests.

The test data should be inspected closely to eliminate errors and inconsistencies. For example, in Fig. 13, the volume change data for the Costa Rica sandy gravel is inconsistent in that the volumetric strains are initially lower for the higher confining pressure. A reasonable interpretation of the data can be seen in Fig. 14.

For the Al-Shawaf - Powell model it is necessary to determine the variation of the parameters G and D with stress. The data from 3 triaxial tests were found not to be sufficient to develop the relationships for G and D. By interpolating between the measured test data, additional curves were developed. This provided a sufficient number of points to develop the required relationships for G and D.

Evaluation of the Bulk Modulus, K, and the Bulk Modulus Factor, f

The bulk modulus factor, K, is evaluated using isotropic triaxial consolidation data. For the Costa Rica sandy gravel triaxial consolidation data was not available for the full stress range of interest, and the data were therefore extrapolated as shown on Fig. 15. At the time the Al-Shawaf - Powell model was being evaluated the effects of membrane penetration on the measured volume changes for this soil were not known. The derivation of the parameter D was accomplished using the value of K calculated from measured volume changes, with no correction for membrane penetration. After realizing that the effects of membrane penetration were quite considerable, as shown on Fig. 15, the variation of K with stress was recalculated. Calculated variations of K for both the measured and the corrected data are shown in Fig. 16.

In order to examine the effect of correcting the measured volume changes for membrane penetration, the stress-strain and volume change

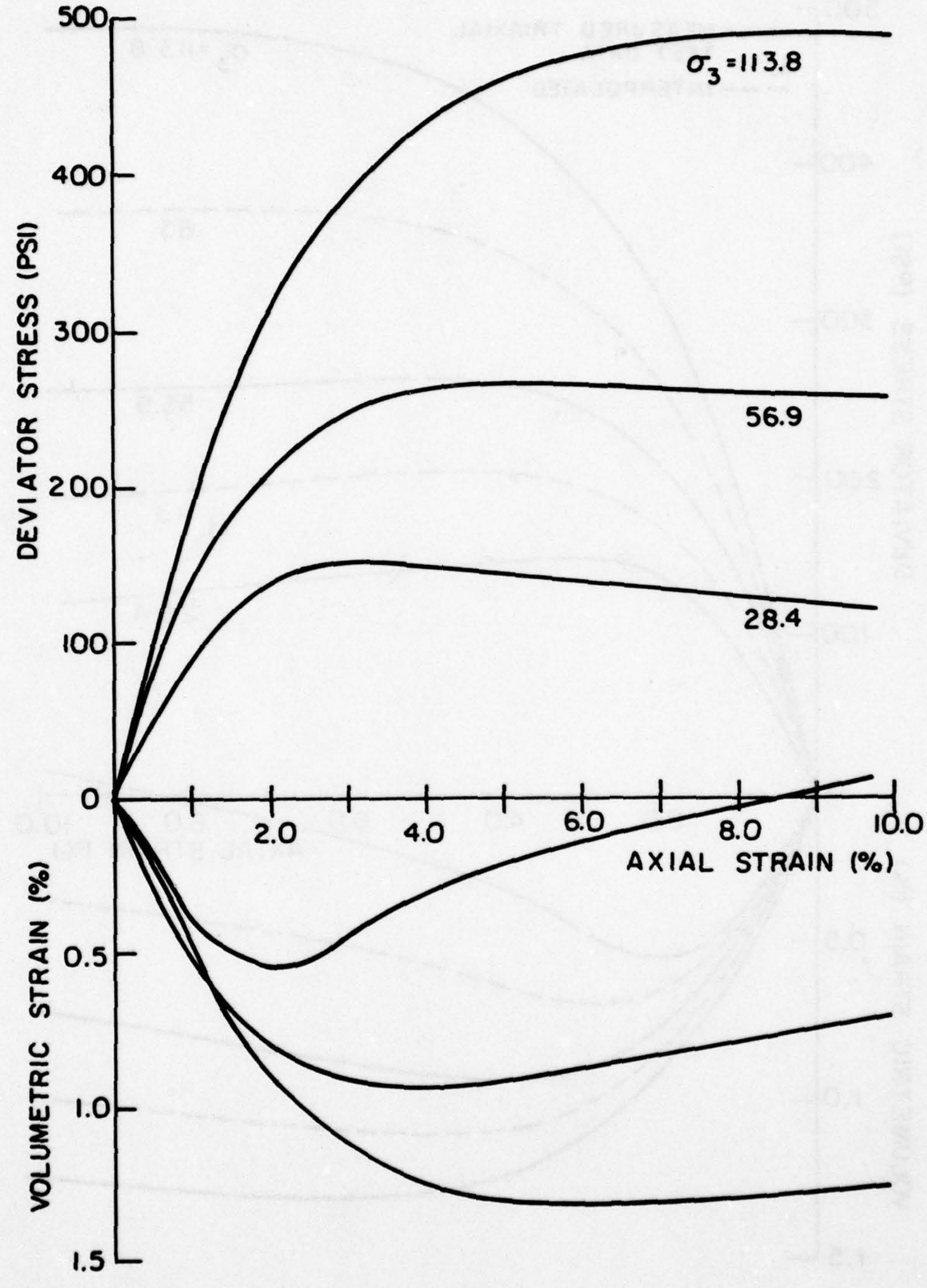


Fig. 13 UNADJUSTED TRIAXIAL TEST DATA FOR THE COSTA RICA SANDY GRAVEL

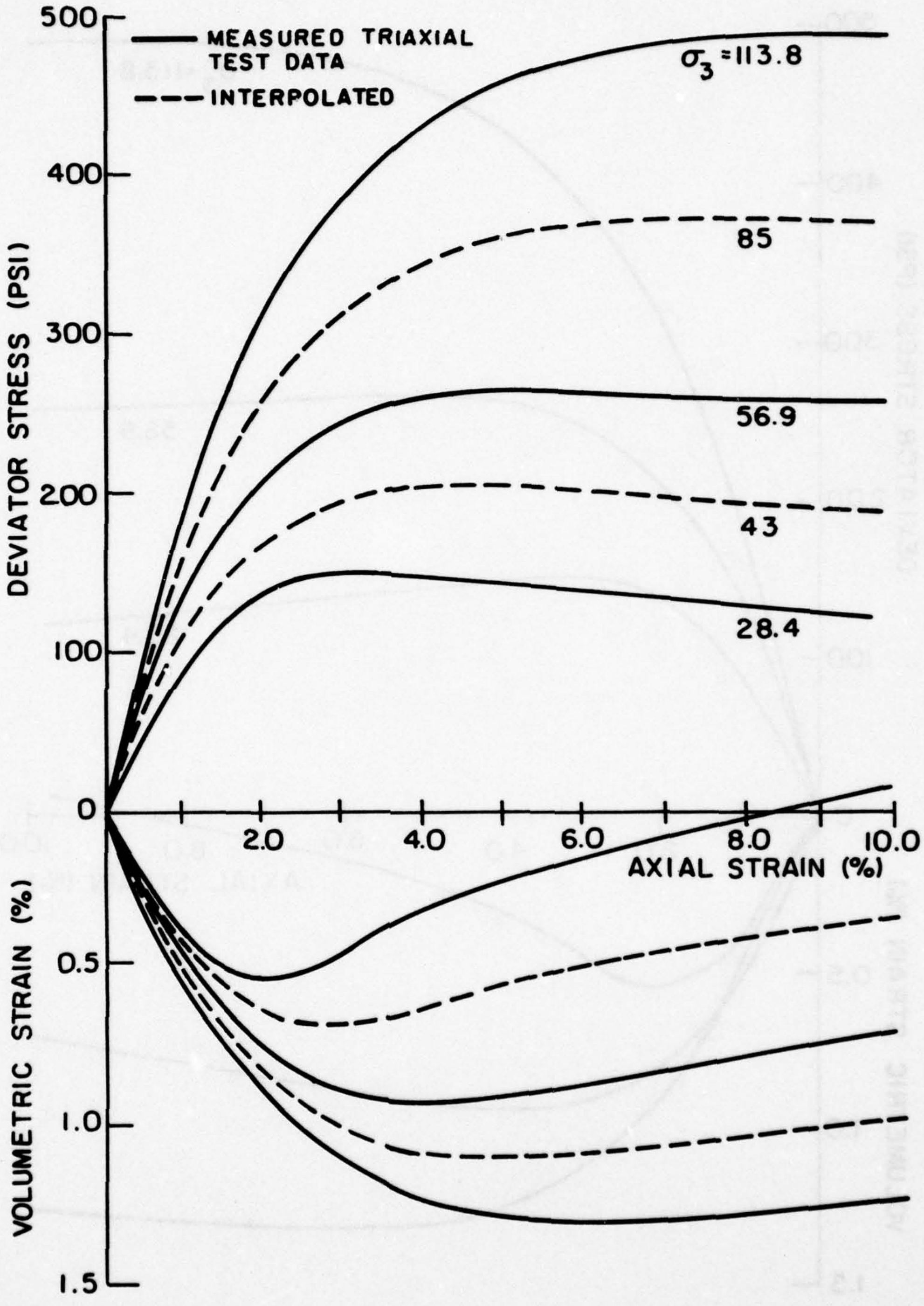


Fig. 14 TRIAXIAL TEST DATA FOR THE COSTA RICA SANDY GRAVEL WITH ADJUSTED VOLUME CHANGE CURVES AND INTERPOLATED CURVES

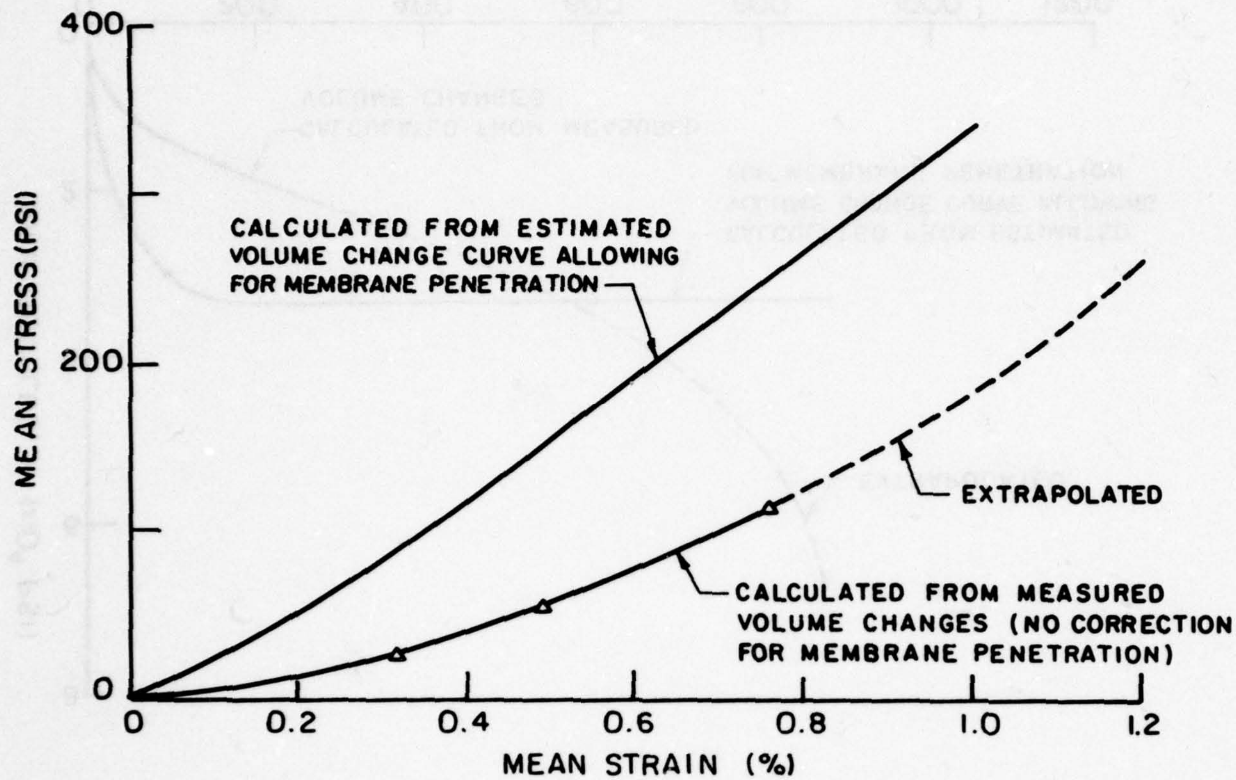


Fig. 15 VARIATION OF MEAN STRESS AND MEAN STRAIN FOR THE COSTA RICA SANDY GRAVEL

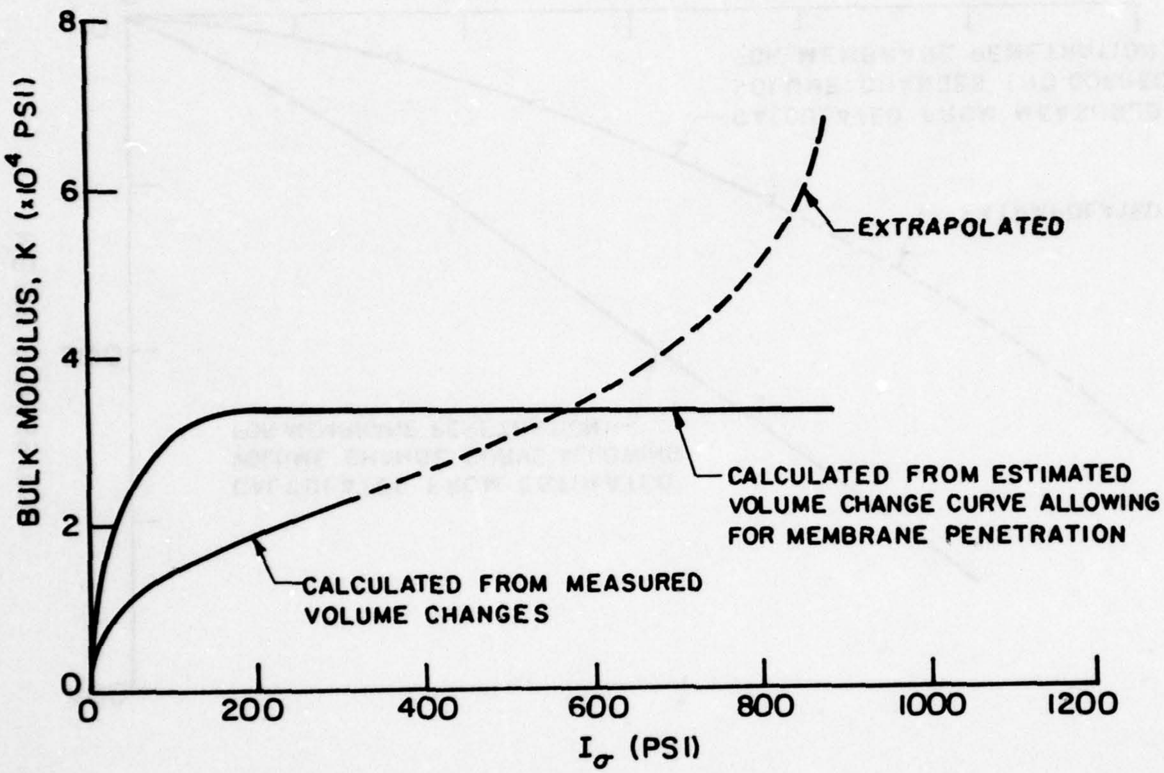


Fig. 16 VARIATION OF BULK MODULUS, K, WITH I_σ FOR THE COSTA RICA SANDY GRAVEL

curves were recalculated for the intermediate confining pressure. Comparing the curves calculated using uncorrected and corrected data showed that the corrections for membrane penetration had these effects:

1. The volume change curve was shifted up
2. The stress-strain curve was not affected.

Based on these results it was decided not to reevaluate all the parameters due to the large amount of work required. There would be no need to reevaluate G since K has no effect on the tangent shear modulus. The value of D would change slightly, but by varying the value of D the curves could be made to provide equally as a good a fit to the corrected data as was accomplished using the values of K with no correction for membrane penetration.

Data for evaluating the unloading bulk modulus was not available for the Costa Rica sandy gravel. Therefore it was necessary to estimate the relationship between the loading and unloading bulk modulus using data for similar soils. The most comprehensive isotropic triaxial consolidation data readily available for cohesionless soils was that for Sacramento River sand, (Lee, 1965). Unloading volume strain data was available for this sand at relative densities of 38% and 100%. Fig. 17 shows the data for relative density of 100%. Average values of the ratio of the unloading modulus to the loading modulus were found to be 3.2 for the 38% relative density and 2.7 for the 100% relative density. Thus an average value of 3 was used for the bulk modulus factor, f.

Evaluation of the Tangent Shear Modulus, G

Two steps are involved in evaluating G. First the triaxial test data must be converted from a stress-strain relationship to a path in the state space. Each point on the triaxial stress-strain curve has a corresponding point in the state space. The values of J and I can be calculated using these equations:

$$J_2^e = \frac{\sqrt{3}}{2} (\varepsilon_a - \frac{\varepsilon_v}{3}) \tag{3}$$

$$I_\sigma = \sigma_1 + 2\sigma_3 \tag{4}$$

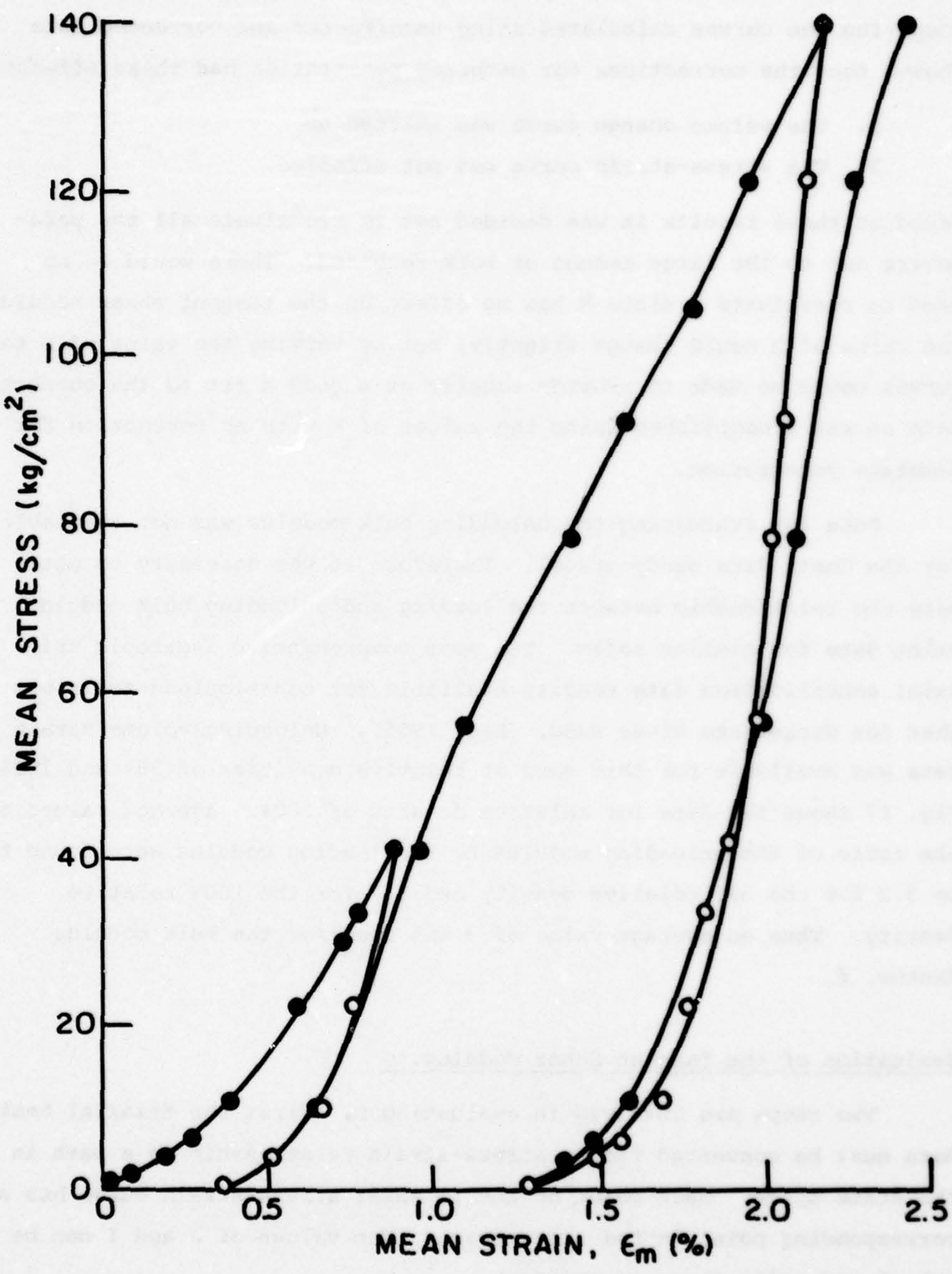


Fig. 17 ISOTROPIC TRIAXIAL CONSOLIDATION TEST DATA FOR DENSE SACRAMENTO RIVER SAND, SHOWING VOLUME CHANGE BEHAVIOR DURING UNLOADING

in which ε_a = axial strain, ε_v = volumetric strain, σ_1 = major principle stress, and σ_3 = minor principle stress.

A sufficient number of points must be calculated to draw smooth curves. The triaxial data for the Costa Rica sandy gravel from Fig. 14 was transposed to the state space as shown on Fig. 18. At this point the spacing and number of J zones must be chosen. While strictly a matter of judgment, if the zones are chosen too large it can be seen that G would vary considerably from zone to zone, and experience shows that this will cause problems in convergence during an analysis.

After the J zones have been chosen, the value of G is then calculated from the slope of the path within a given J zone using the following equation

$$G = \frac{1}{2\sqrt{3}} \frac{\Delta I_\sigma}{\Delta J_2^e} \tag{5}$$

in which ΔI_σ = change in I_σ as the path crosses from one J zone to the next, and ΔJ_2^e = change in J_2^e as the path crosses from one J zone to the next.

One value of G is calculated for each stress path within each zone. The calculated value of G is then plotted against the average value of I_σ for the increment as shown on Fig. 19.

Evaluation of the Tangent Value of the Dilatancy Ratio, D

The evaluation of the parameter D involves several steps. The first step is to convert the volume change behavior from the triaxial tests to its equivalent curve in the Al-Shawaf - Powell model. Each point on the volumetric curve has a corresponding point with coordinates ε_m and J_2^e, which can be calculated using the following equations:

$$\varepsilon_m = \text{Volumetric Strain}/3 \tag{6}$$

$$J_2^e = \frac{\sqrt{3}}{2} \left(\varepsilon_a - \frac{\varepsilon_v}{3}\right) \tag{7}$$

The relationship between ε_m and J_2^e for the Costa Rica sandy

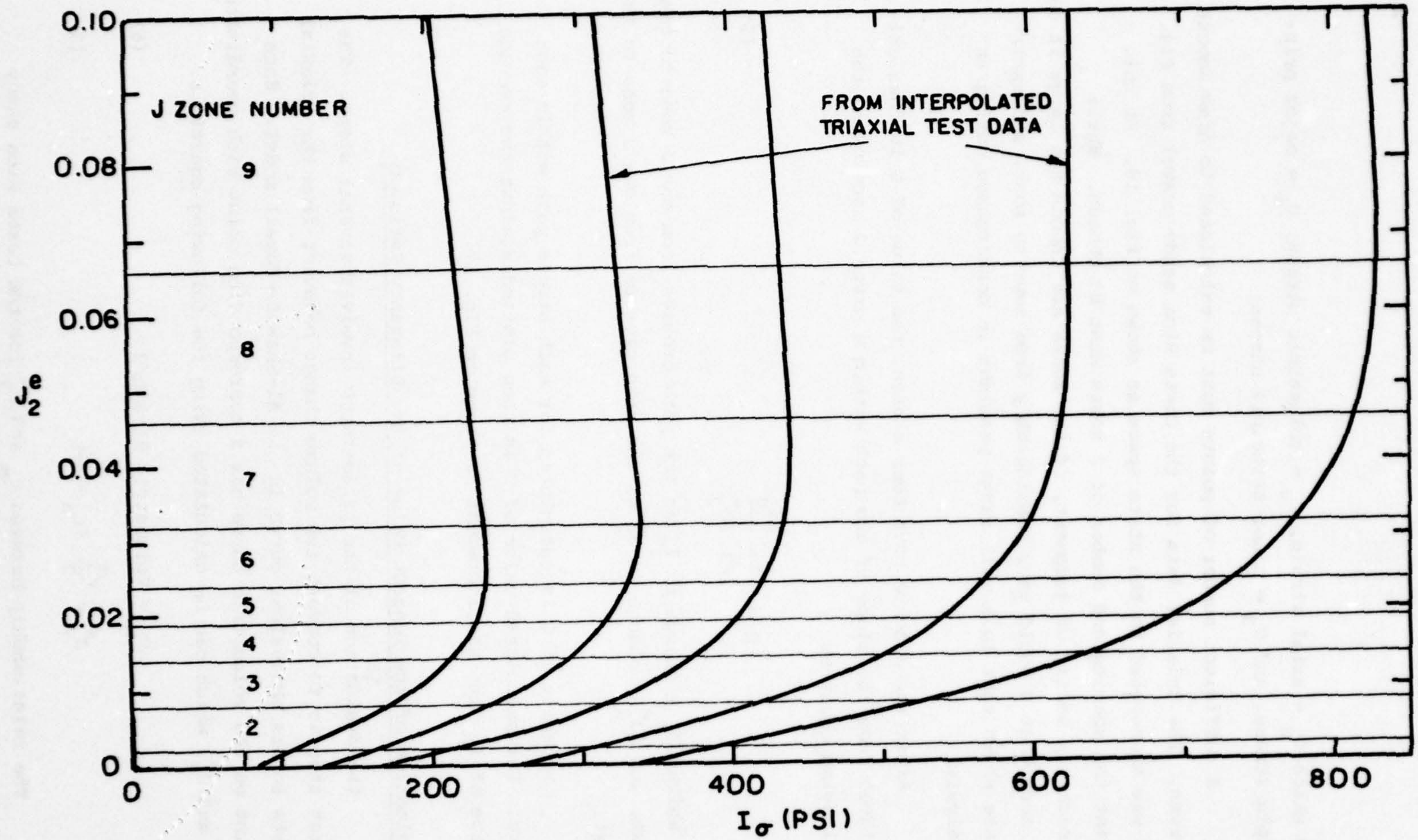


Fig. 18 TRIAXIAL TEST DATA FOR THE COSTA RICA SANDY GRAVEL PLOTTED IN THE AL-SHAWAF - POWELL STATE SPACE

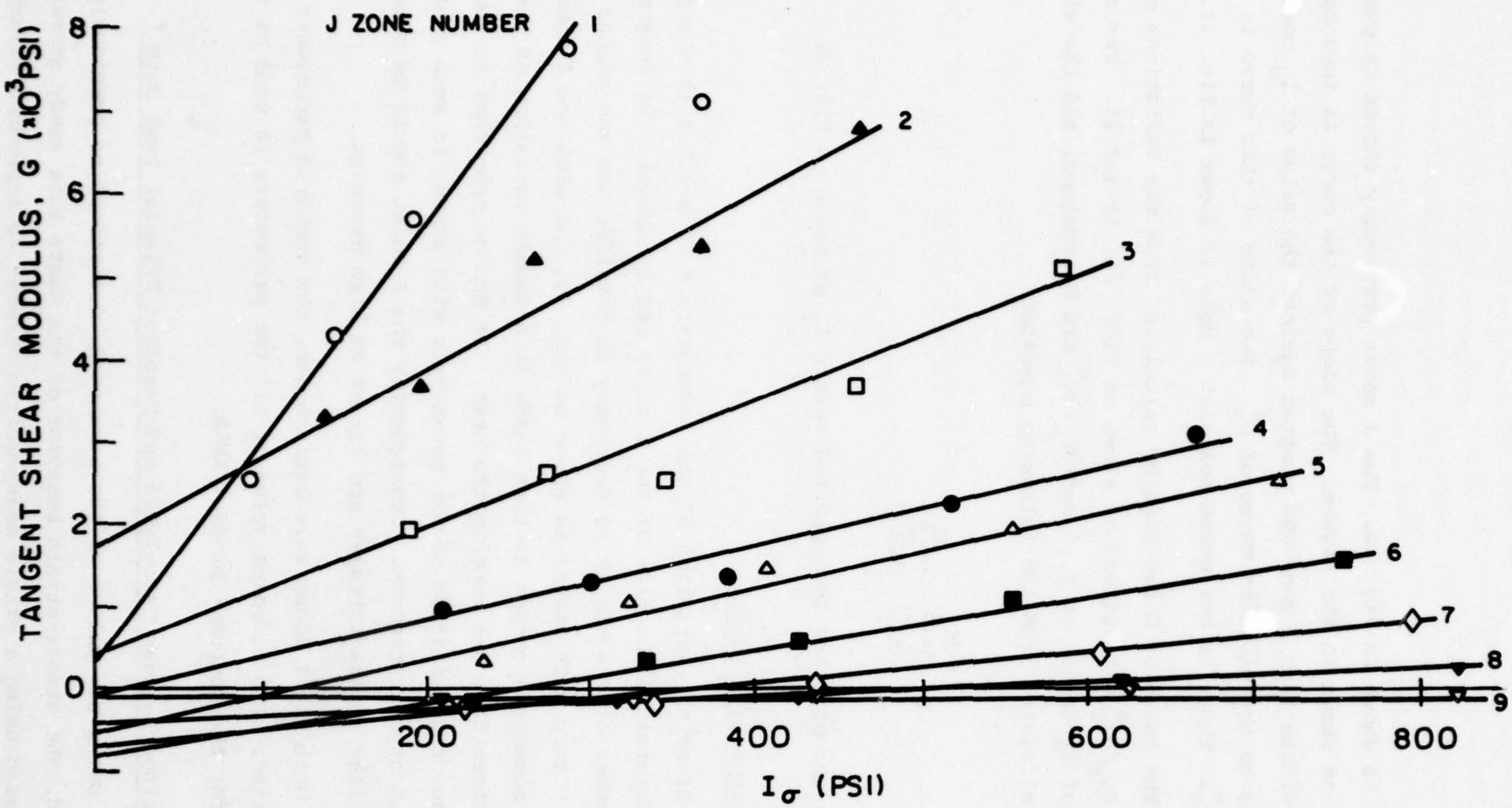


Fig. 19 VARIATION OF THE TANGENT SHEAR MODULUS, G, WITH I_σ FOR DIFFERENT J ZONES FOR THE COSTA RICA SANDY GRAVEL

gravel is shown in Fig. 20. The J zones previously chosen in evaluating G are shown in the figure. The slope of the curve is then calculated within each J zone and plotted against the value of I_σ corresponding to the midpoint value of J_2^e. The slope of this curve is $d\varepsilon_m/dJ_2^e$. There is one curve for each J zone as shown in Fig. 21.

The value of D can then be calculated from the variations of K, G and $d\varepsilon_m/dJ_2^e$ with stress as shown on Figs. 16, 19 and 21. For a given value of I_σ values of K, G and $d\varepsilon_m/dJ_2^e$ are determined, and the value of D is calculated from the following equation:

$$D = \frac{d\varepsilon_m}{dJ_2^e} - \frac{2G}{3\sqrt{3}\,K} \tag{8}$$

The values of D are then plotted versus I_σ as shown on Fig. 22.

Evaluation of I Zones

After the variation of the parameters K, G and D with stress has been determined the size of the I zones can be chosen. To determine the number of I zones it is necessary to consider how the values of K, G and D vary with stress as shown on Fig. 23. As with the J zones, the I zones are chosen so that there is a smooth variation of the parameters from one zone to the next. It can be seen that at low stresses the variation of the parameters with stress is much greater than at higher stresses. Consequently the I zones should be chosen to be smaller at low stresses and larger at high stresses.

Once the I zones have been chosen, the required parameters can be tabulated. The midpoint value of all the parameters is used as input data for the computer program ANSR.

Comparison between Calculated and Measured Triaxial Test Data

Using the values of the parameters determined as previously discussed, the stress-strain behavior of the Costa Rica sandy gravel was simulated using a single axisymmetric element to reproduce triaxial test behavior. The behavior was calculated for three different

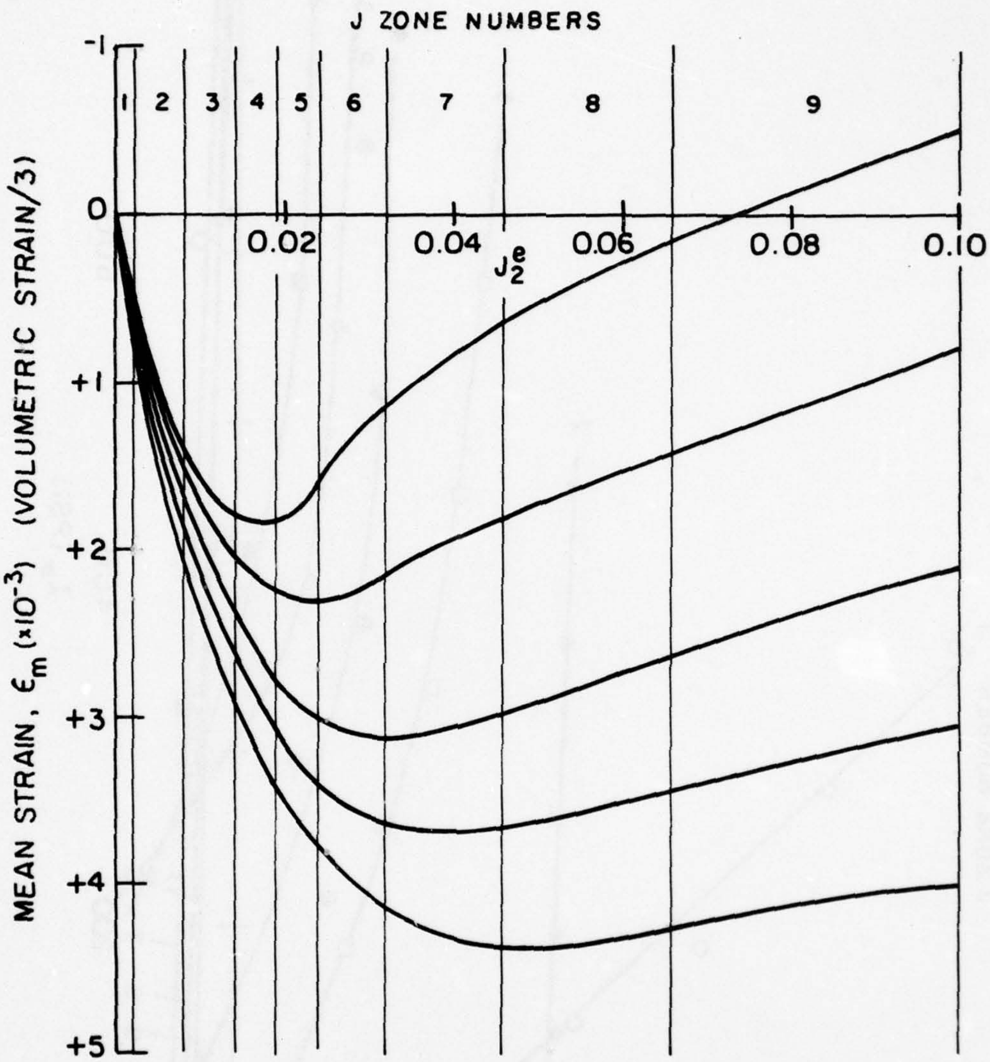


Fig. 20 VARIATION OF MEAN STRAIN, ε_m, WITH J_2^e FOR THE COSTA RICA SANDY GRAVEL

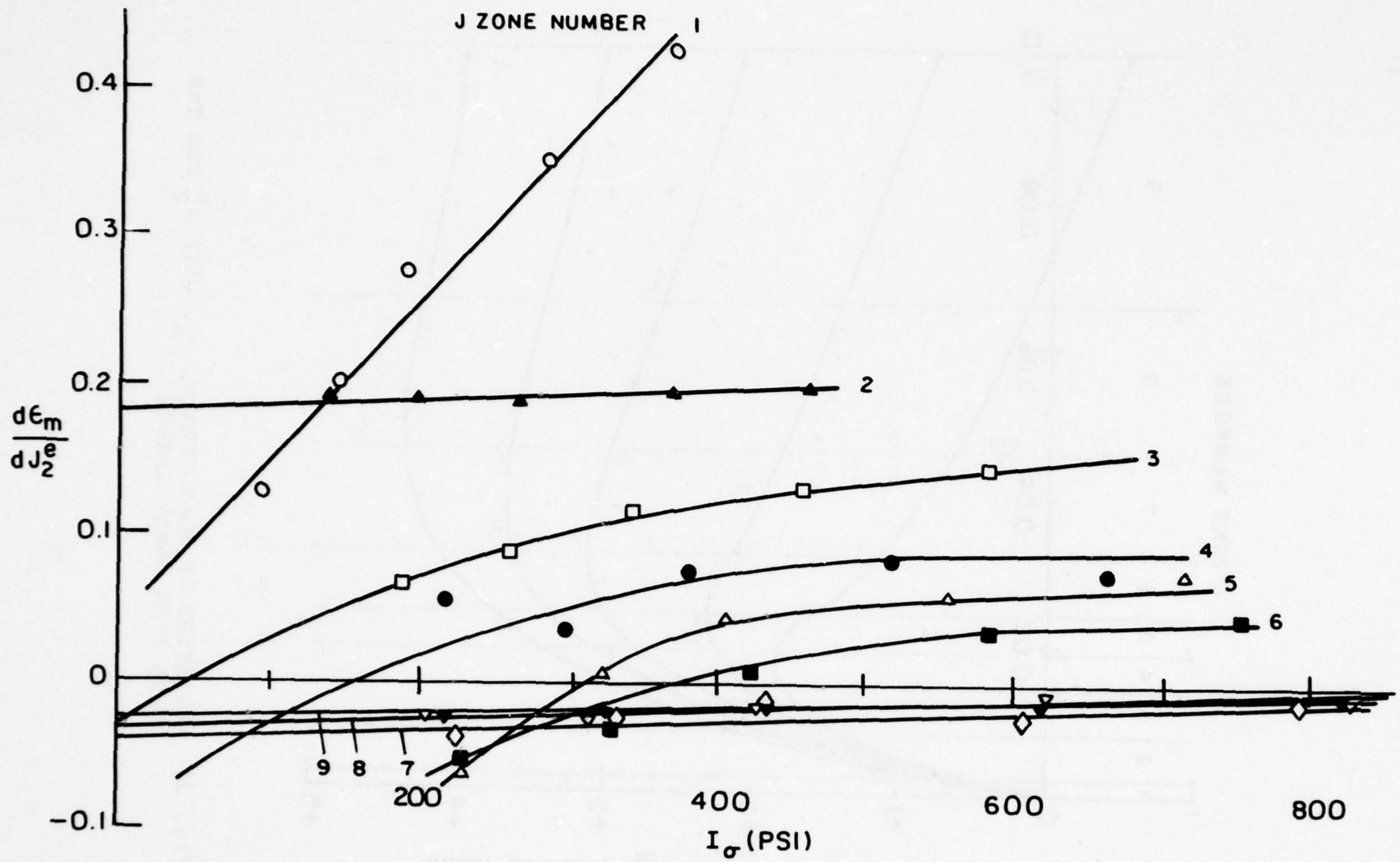


Fig. 21 VARIATION OF THE SLOPE OF THE MEAN STRAIN VERSUS J CURVES WITH I_σ FOR EACH J ZONE FOR THE COSTA RICA SANDY GRAVEL

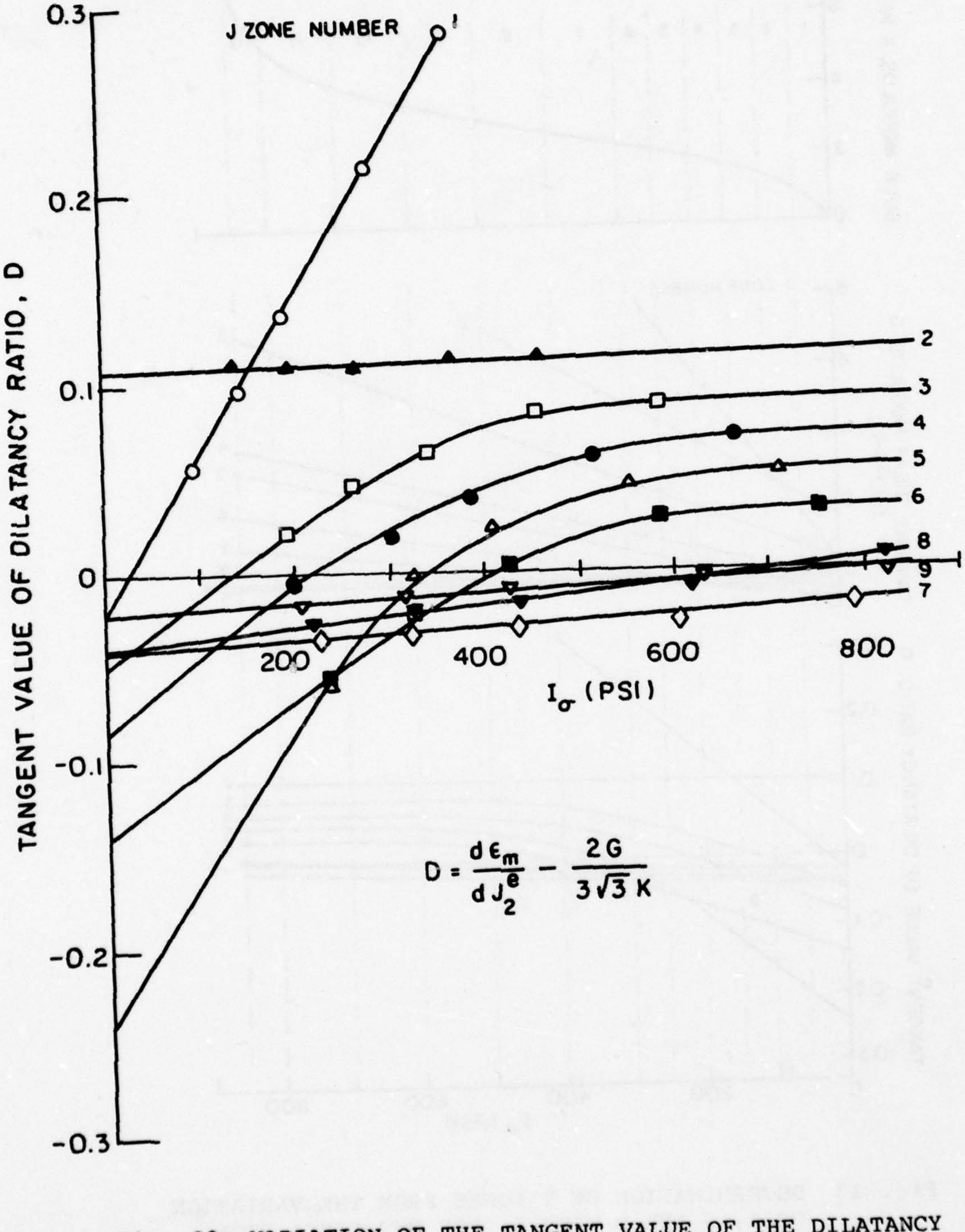


Fig. 22 VARIATION OF THE TANGENT VALUE OF THE DILATANCY RATIO, D, WITH I_σ FOR THE COSTA RICA SANDY GRAVEL

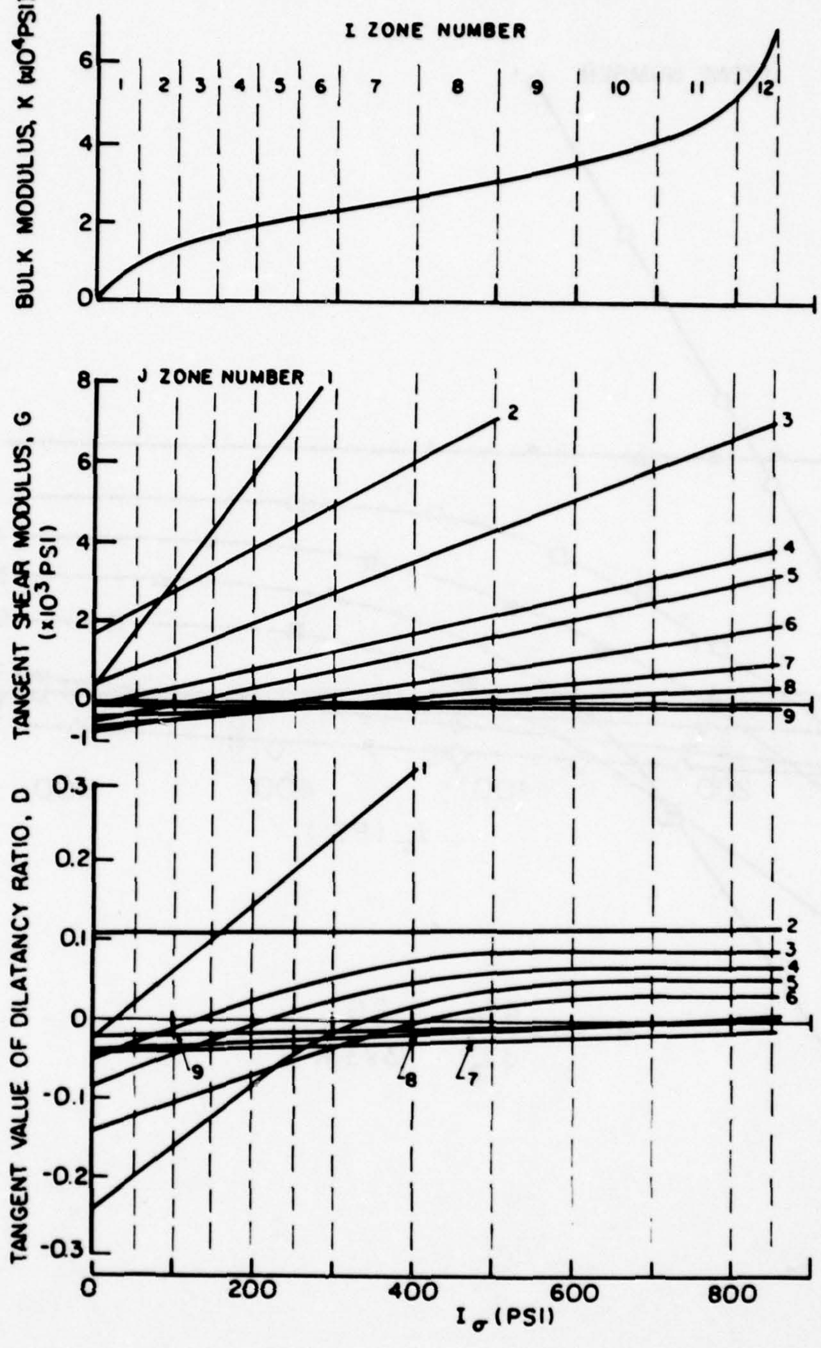


Fig. 23 DETERMINATION OF I ZONES FROM THE VARIATION OF K, G AND D WITH I_σ FOR THE COSTA RICA SANDY GRAVEL

confining pressures as shown in Fig. 24. As can be seen in this figure, the calculated data closely approximate the measured data.

The curves can be adjusted to more exactly match the measured data by changing the values of G and D slightly as previously discussed. By following the path in the state space and determining which zones the path crosses, appropriate changes in the parameters can be made. For example, the calculated stress-strain curve for the high confining pressure was slightly above the measured curve; by reducing the value of G slightly in several of the zones, the curve was adjusted to fit as shown in Fig. 25. Adjustment of the high confining pressure curve had no affect on the other two curves, since they followed different paths in the state space. No adjustment was made for the curves for the middle and low confining pressures.

The calculated volume change curve for the intermediate confining pressure was slightly higher than the experimental curve. Adjustment of the values of D corresponding to axial strains between 2.5% and 4% axial strain brought the entire curve down as shown on Fig. 25. The calculated volumetric strain curve for the highest confining pressure was slightly high, and at axial strains greater than 6.5% the values of D were of the wrong sign as shown by the fact that the calculated curve indicated the volume change behavior was going from dilative to contractive behavior. This was corrected by increasing the values of D corresponding to axial strains between 4% and 6%, and decreasing the values of D for larger strains.

After several attempts to determine the characteristics of the Al-Shawaf - Powell model in plane strain, it was learned that additional laboratory data is required for this case. To model other than triaxial compression conditions the Al-Shawaf - Powell model requires data from triaxial extension tests. These data must be plotted in the state space as shown in Fig. 26. The spacing of the J zones must then be determined for the triaxial extension data and compared to the spacing of the J zones for triaxial compression data so that the ratio:

$$.5 \leq \frac{J_2^e \text{ extension}}{J_2^e \text{ compression}} = \text{constant} \leq 1.0 \tag{9}$$

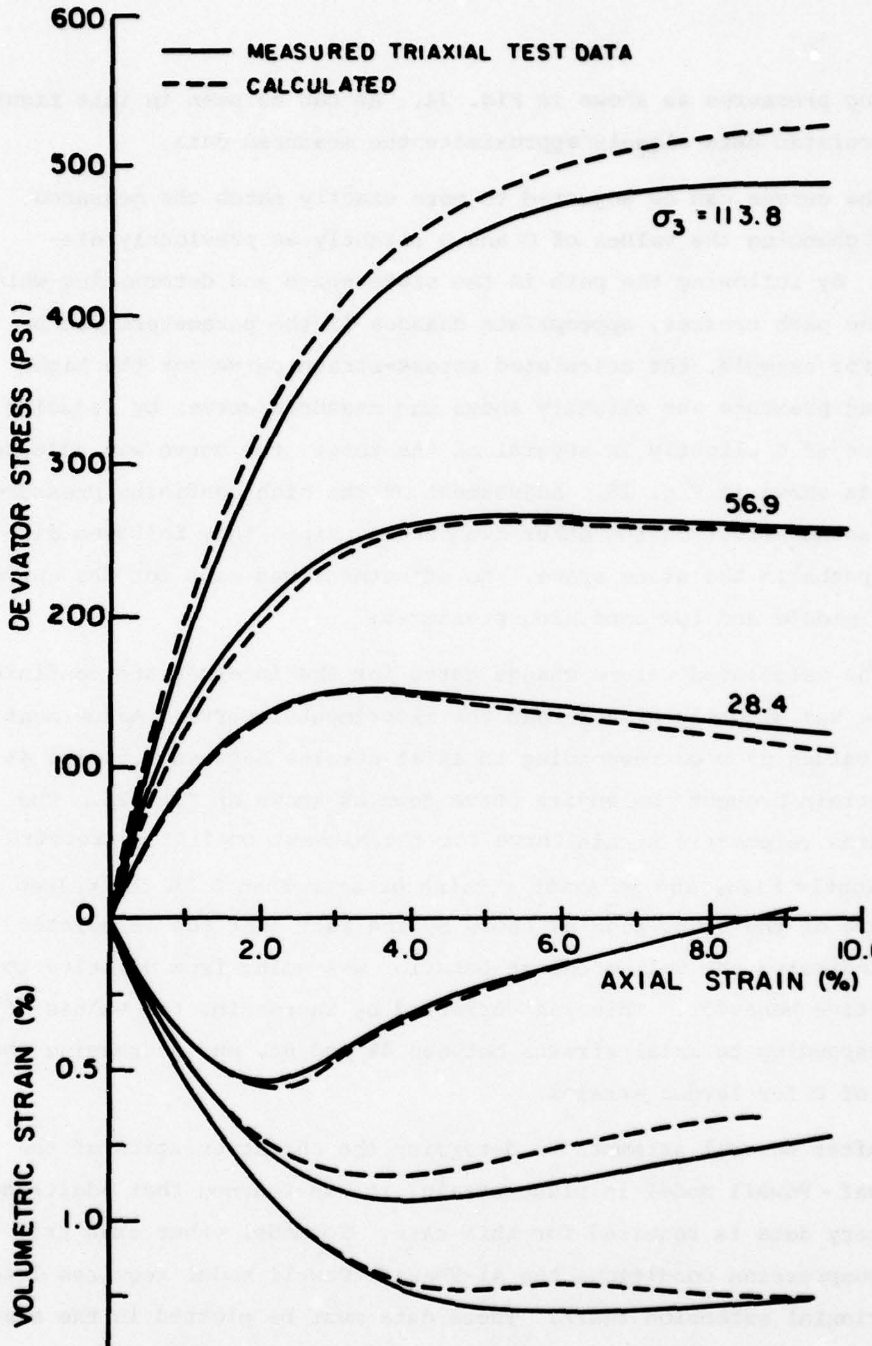


Fig. 24 COMPARISON OF MEASURED TRIAXIAL STRESS-STRAIN AND VOLUME CHANGE CURVES WITH THOSE CALCULATED USING UNADJUSTED DATA FOR THE AL-SHAWAF - POWELL MODEL

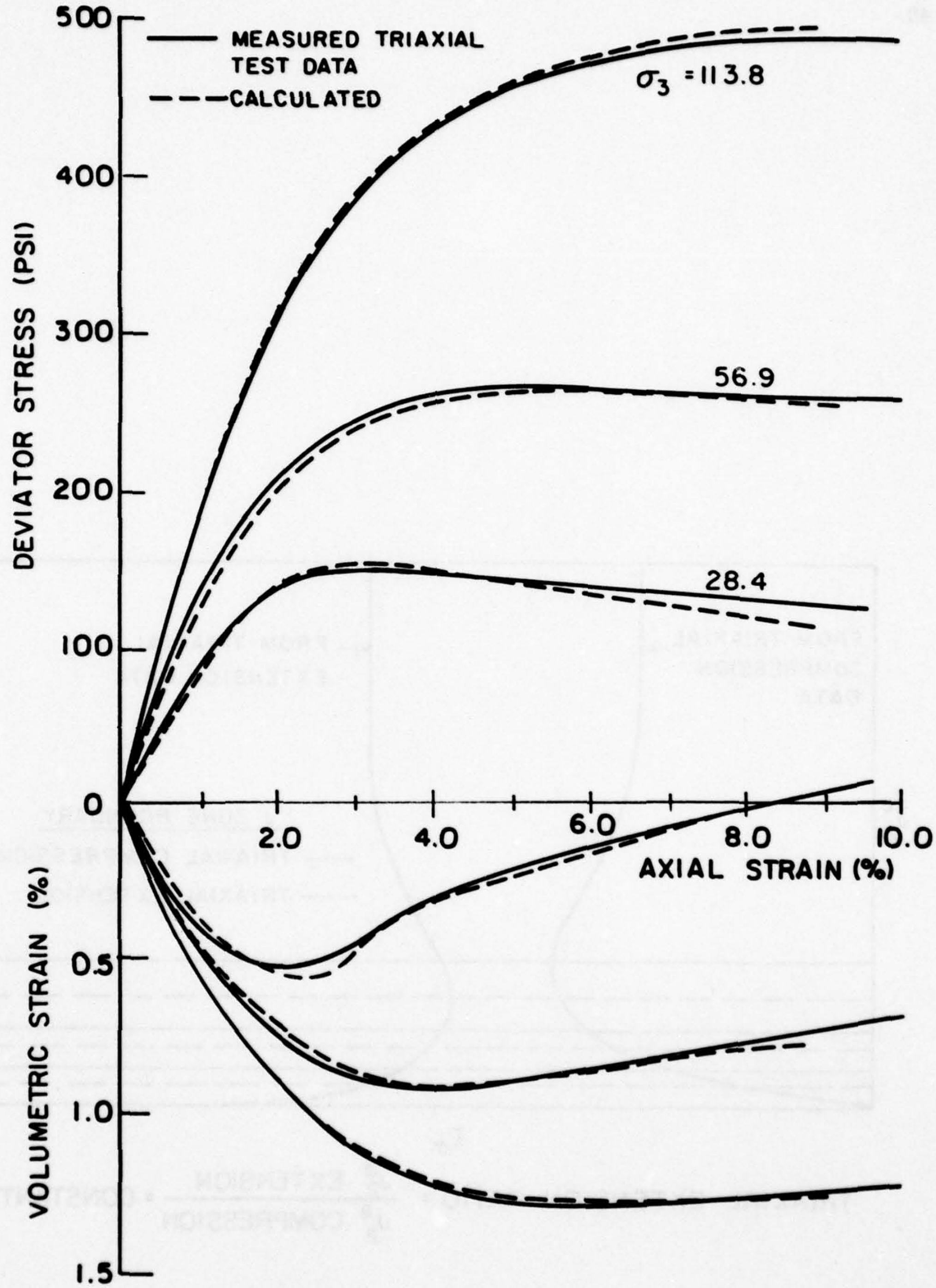
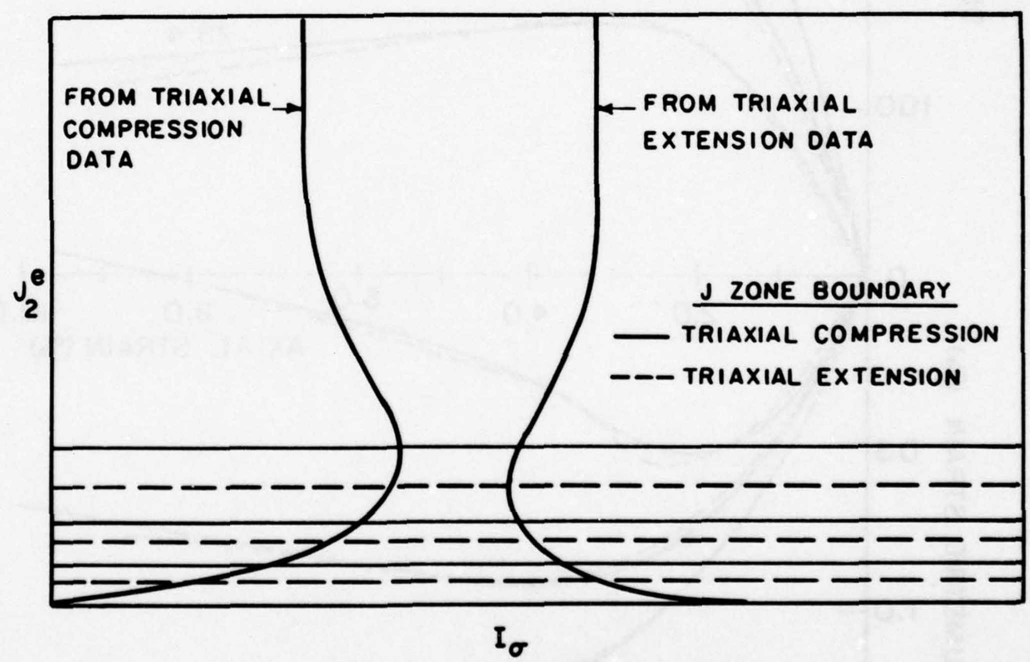


Fig. 25 COMPARISON OF MEASURED STRESS-STRAIN AND VOLUME CHANGE CURVES WITH THOSE CALCULATED USING ADJUSTED VALUES OF G AND D FOR THE COSTA RICA SANDY GRAVEL



$$\text{TRIAXIAL EXTENSION RATIO} = \frac{J_2^e \text{ EXTENSION}}{J_2^e \text{ COMPRESSION}} = \text{CONSTANT}$$

Fig. 26 COMPARISON OF TRIAXIAL COMPRESSION AND EXTENSION TEST DATA IN THE STATE SPACE

This ratio is called the triaxial extension ratio. The range of .5 to 1.0 is suggested by Al-Shawaf. There is no data available to make an estimate of the triaxial extension ratio for the Costa Rica sandy gravel. Due to the relatively high computer costs associated with the Al-Shawaf - Powell model and the fact that the characteristics of the model had already been evaluated for triaxial compression, it was considered unnecessary to study the plane strain condition.

Summary of the Characteristics of the Al-Shawaf - Powell Model

Based on the examination of the characteristics of the Al-Shawaf - Powell model for axisymmetric conditions described previously, the following conclusions were drawn:

1. The model offers an advantage in that shear dilation effects can be modelled, and it has been shown that the model can reproduce the behavior of triaxial tests from which the parameters are calculated. The behavior of the model under plane strain conditions has not been studied.

2. To calculate the required parameters requires about 4 to 5 working days for an engineer experienced with the process. If, for example, it was necessary to study a range of stress-strain behavior, the time required to develop the parameters for the upper and lower bounds required would be about two weeks.

3. The parameters are not related to soil behavior in a manner that allows an engineer to develop a "feel" for what values are reasonable for a given type of soil. With the numerical technique, there is no simple means of comparing the values of the parameters for various soils or soil types. The parameters are only related to a given zone in the state space, and their values depend on the size of the zone as well as the characteristics of the soil.

4. The parameters require the following laboratory tests:
   1. Triaxial compression
   2. Triaxial extension

3. Triaxial consolidation

While triaxial compression and consolidation test data are frequently available, triaxial extension test data are not. It is not known how many triaxial extension tests would be required to determine the value of the triaxial extension ratio.

CAM CLAY MODEL

The Cam Clay model is an elastic-plastic work hardening model originally developed at Cambridge by Roscoe and his co-workers. A recent study by Chang and Duncan (1977) has proposed several modifications to the model in order to provide a better model for the stress-strain and volume change behavior of compacted materials used in earth or rockfill dams.

The modifications proposed by Chang and Duncan are as follows:

1. The formulation was generalized to include a failure line with a cohesion intercept.
2. The relationship was modified so that the measured void ratio--effective stress relationship could be used, rather than a straight-line approximation.
3. The failure criterion was changed to accommodate a curved failure envelope to account for particle crushing at high confining pressures for cohesionless soils.
4. The parameter α was added. This parameter determines the magnitude of the plastic shear strains.

The theoretical developments of the above modifications were presented by Chang and Duncan (1977) and will not be discussed in this report. The procedures for developing the calculated parameters and the results of a parametric study are discussed in the following sections, and the calculated stress-strain and volume change behavior using the computer program MODEL is compared to measured data for three soils.

Development of the Calculated Parameters for the Cam Clay Model

Evaluation of ϕ_o and $\Delta\phi$ for Cohesionless Soils

The Mohr Envelopes for almost all soils are curved to some extent, and the wider the range of pressures involved, the greater the curvature. In the case of cohesionless soils, such as sands, gravels and rockfills,

this curvature makes it difficult to select a single value of ϕ which is representative of the full range of pressures of interest. For example, in the bottom near the center of a large dam, rockfill may be confined under such large pressures that the friction angle may be several degrees smaller than near the surface of the slopes. If a value of ϕ is selected which is appropriate for the center of the dam, it will be too small to represent the strength of the material near the slopes, and if a value appropriate for the slopes is selected it will be too large to represent the strength of the material near the center of the dam.

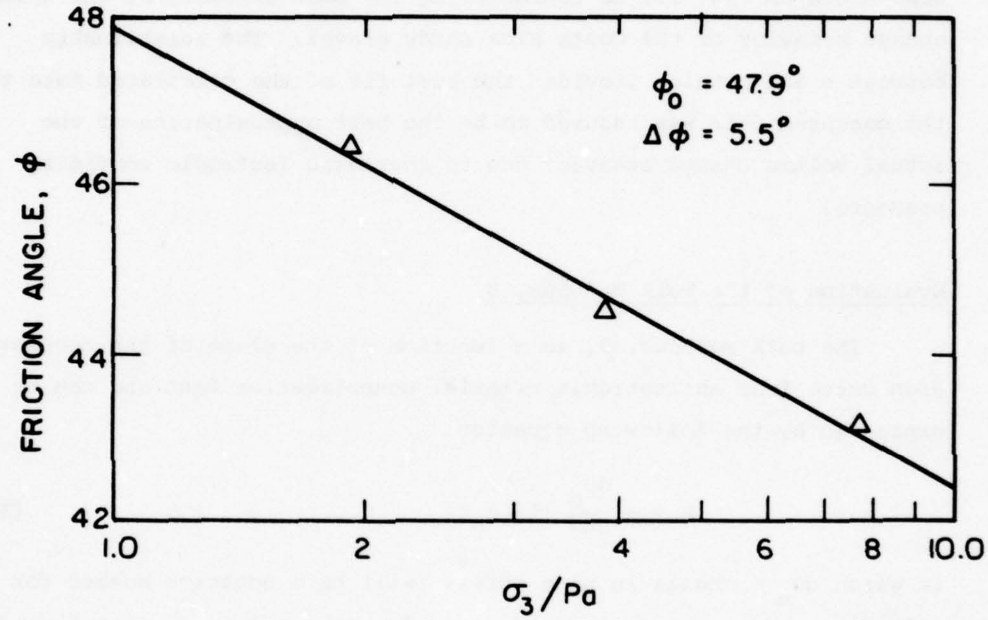
One means of circumventing the difficulties involved in the use of a single value of ϕ is to use values of ϕ for the material which vary with confining pressure. For a given series of tests, if it is assumed that the envelope for each test passes through the origin, the variation of ϕ can be expressed as shown on Fig. 27. The variation shows on Fig. 27 can be represented by an equation of the form

$$\phi = \phi_o - \Delta\phi \log_{10}\left(\frac{\sigma_3}{p_a}\right) \tag{10}$$

In this equation ϕ_o is the value of ϕ for σ_3 equal to one atmosphere, and $\Delta\phi$ is the reduction in ϕ for a 10-fold increase in σ_3. For this particular example the data for the Costa Rica sandy gravel is used.

Evaluation of the Relationship between Void Ratio (e) and Isotropic Confining Pressure (P)

The relationship between void ratio and isotropic confining pressure may be developed from isotropic triaxial consolidation tests. Standard one-dimensional consolidation tests can also be used but the value of the confining stress (σ_3) must be estimated in this case. In order to determine the isotropic confining pressure for a given void ratio the value of the coefficient of earth pressure at rest (K_o) must be assumed. If triaxial data are used, the value of K_o need not be assumed. The volume changes measured in triaxial tests are, however, subject to inaccuracies resulting from membrane penetration of unknown magnitudes in coarse-grained soils.



σ_3	σ_3/P_a	$(\sigma_1-\sigma_3)_f$	$(\sigma_1+\sigma_3)_f$	$\sin^{-1}\left(\frac{\sigma_1-\sigma_3}{\sigma_1+\sigma_3}\right)_f$
28.4	1.93	151	208	46.5
56.9	3.87	268	382	44.5
113.8	7.74	491	718	43.1

Fig. 27 VARIATION OF FRICTION ANGLE WITH CONFINING PRESSURE FOR THE COSTA RICA SANDY GRAVEL

The variation of void ratio with isotropic confining pressure for the Costa Rica sandy gravel is shown on Fig. 28. Because the measured volume change data include volume changes due to membrane penetration, volume changes were reduced in a series of trials until the relationship was found which provided the best fit to triaxial data. This volume change relationship is shown in Fig. 28 and was also shown on Fig. 15, as representing the best estimate of the volume change behavior of the Costa Rica sandy gravel. The relationship between e and p which provided the best fit of the calculated data to the measured data was assumed to be the best approximation of the actual volume change behavior due to increased isotropic confining pressure.

Evaluation of the Bulk Modulus, B

The bulk modulus, B, is a function of the slope of the recompression curve from an isotropic triaxial consolidation test and can be expressed by the following equation

$$B = -\frac{d\sigma_m}{de}(1+e_o) \tag{11}$$

in which $d\sigma_m$ = change in mean stress (will be a negative number for unloading), de = change in void ratio for $d\sigma_m$, and e_o = initial void ratio.

As can be seen, B represents a linear approximation of the recompression behavior over the range of $d\sigma_m$. In the Cam Clay model the slope of the recompression curve is a constant, and does not vary with stress as does the slope of loading curve. This can cause problems if, for example, as shown on Fig. 29, the rebound portion of the test is taken between points such as A and B. Because the same slope will be used at all pressures, including pressures greater than at B, this slope will be used at point C, resulting in a recompression curve given by line C-D, and the rebound curve will be steeper than the compression curve. The plastic strains are a function of the difference in slopes of the compression and recompression curve, for a recompression curve with a slope from C to D the plastic strains will have the wrong sign. This will lead to a reverse

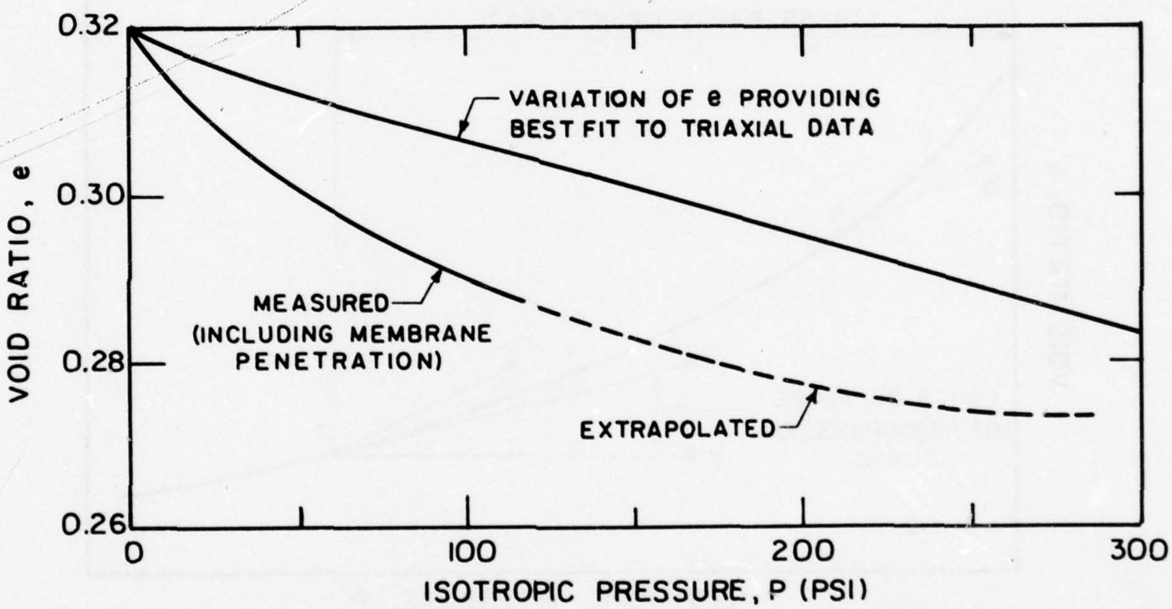


Fig. 28 VARIATION OF VOID RATIO, e, WITH ISOTROPIC PRESSURE, P, FOR THE COSTA RICA SANDY GRAVEL

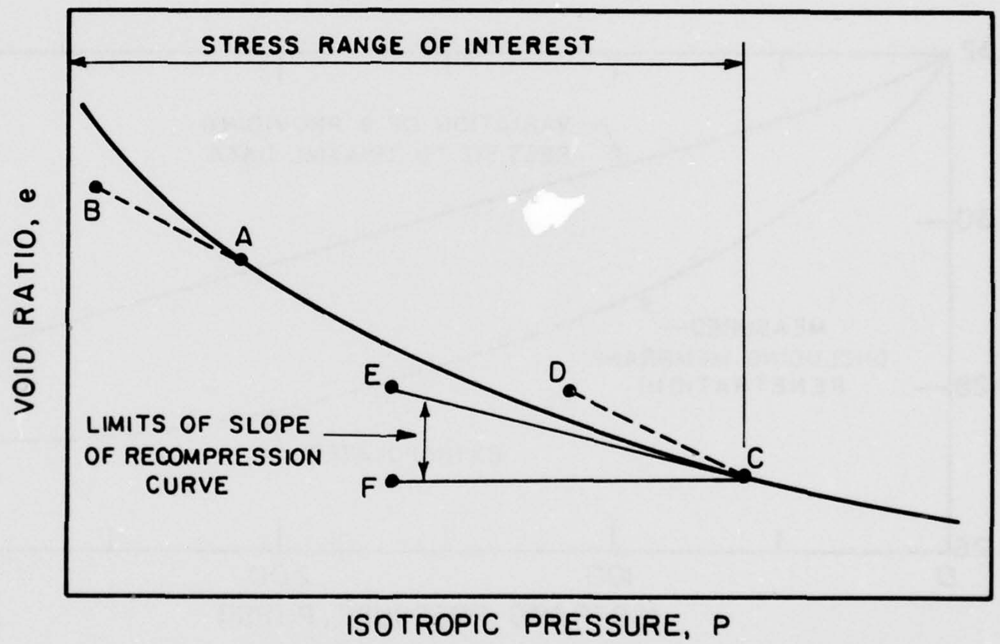


Fig. 29 LIMITATIONS ON SLOPE OF RECOMPRESSION CURVE FOR THE CAM CLAY MODEL

curvature of the calculated stress-strain curve, resulting in decreasing strain with increasing stress.

To avoid this problem it is necessary to consider the range of interest for the isotropic stress. If, as shown on Fig. 29, the range of interest extends to point C, the practical range of values for the slope of the recompression curve will be within the limits defined by points E and F. Points E and C define a line tangent to the compression curve at point C, and points F and C define a horizontal line through point C.

Effect of a Shift in the Void Ratio - Isotropic Pressure Curve on the Calculated Stress-Strain and Volume Change Behavior

As previously discussed, membrane penetration can produce measured volume changes which are considerably larger than those representative of the actual behavior of the soil. The method used in this study to correct the volume changes was to use repeated trials to determine the e-p curve which provided the best fit to the measured triaxial stress-strain data.

The affect of shifting the e-p curve is shown on Fig. 30. As the e-p curve is flattened, the calculated stress-strain curve becomes steeper. Shifting the e-p curve does not affect the value of the deviator stress at failure, however, because that is a function only of ϕ and $\Delta\phi$. The volume change curve is shifted up as the e-p curve is flattened, as would be expected.

Effect of a Change in the Slope of the Recompression Curve on the Calculated Stress-Strain and Volume Change Behavior

A change in the slope of the recompression curve affects the stress-strain and volume change behavior as shown on Fig. 31. Shifting the recompression curve from A to B results in larger plastic strains and smaller elastic strains. The plastic strains are a function of the difference in slope between the compression and recompression curves, as previously discussed. The effect of flattening the slope of the recompression curve is to increase the difference in slope between the

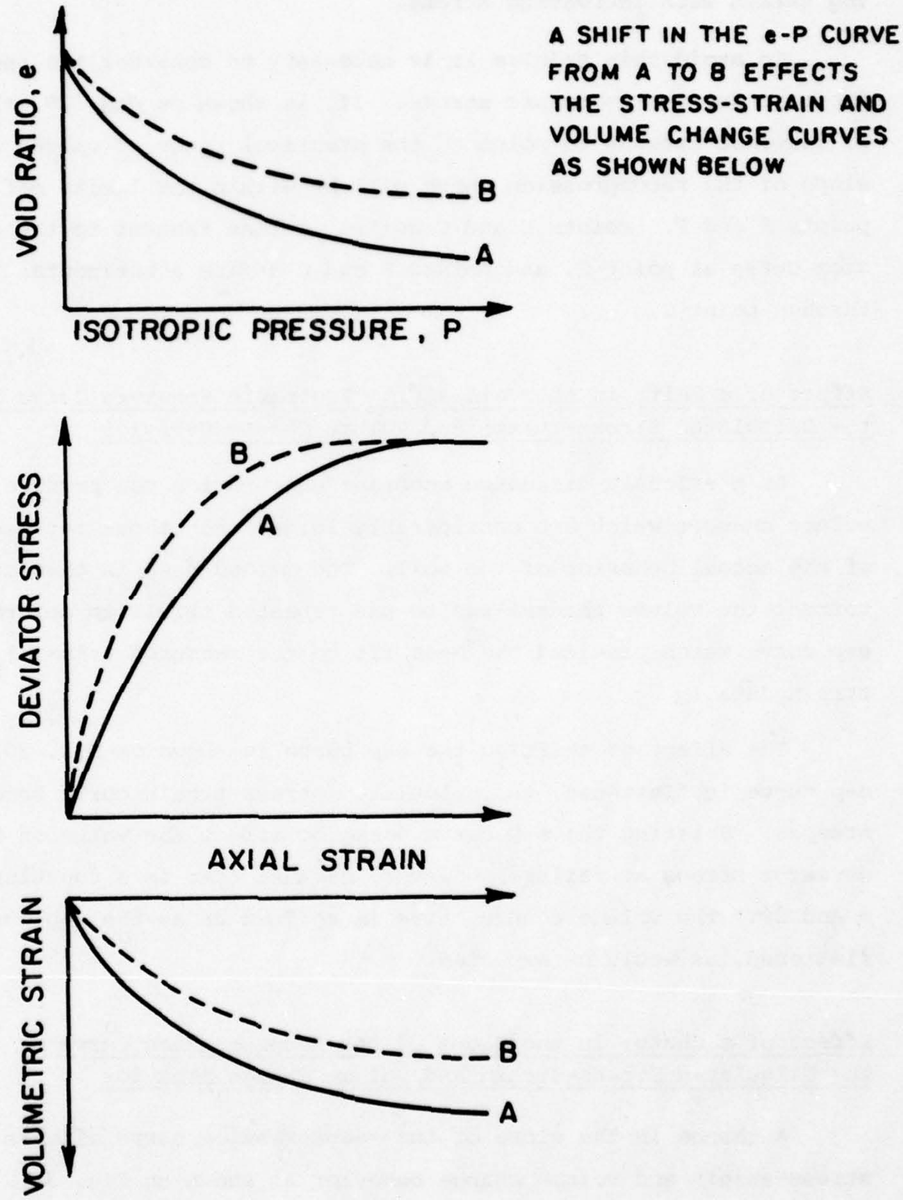


Fig. 30 EFFECT OF A SHIFT IN THE VOID RATIO - ISOTROPIC PRESSURE RELATIONSHIP ON THE STRESS-STRAIN AND VOLUME CHANGE CURVES CALCULATED USING THE CAM CLAY MODEL

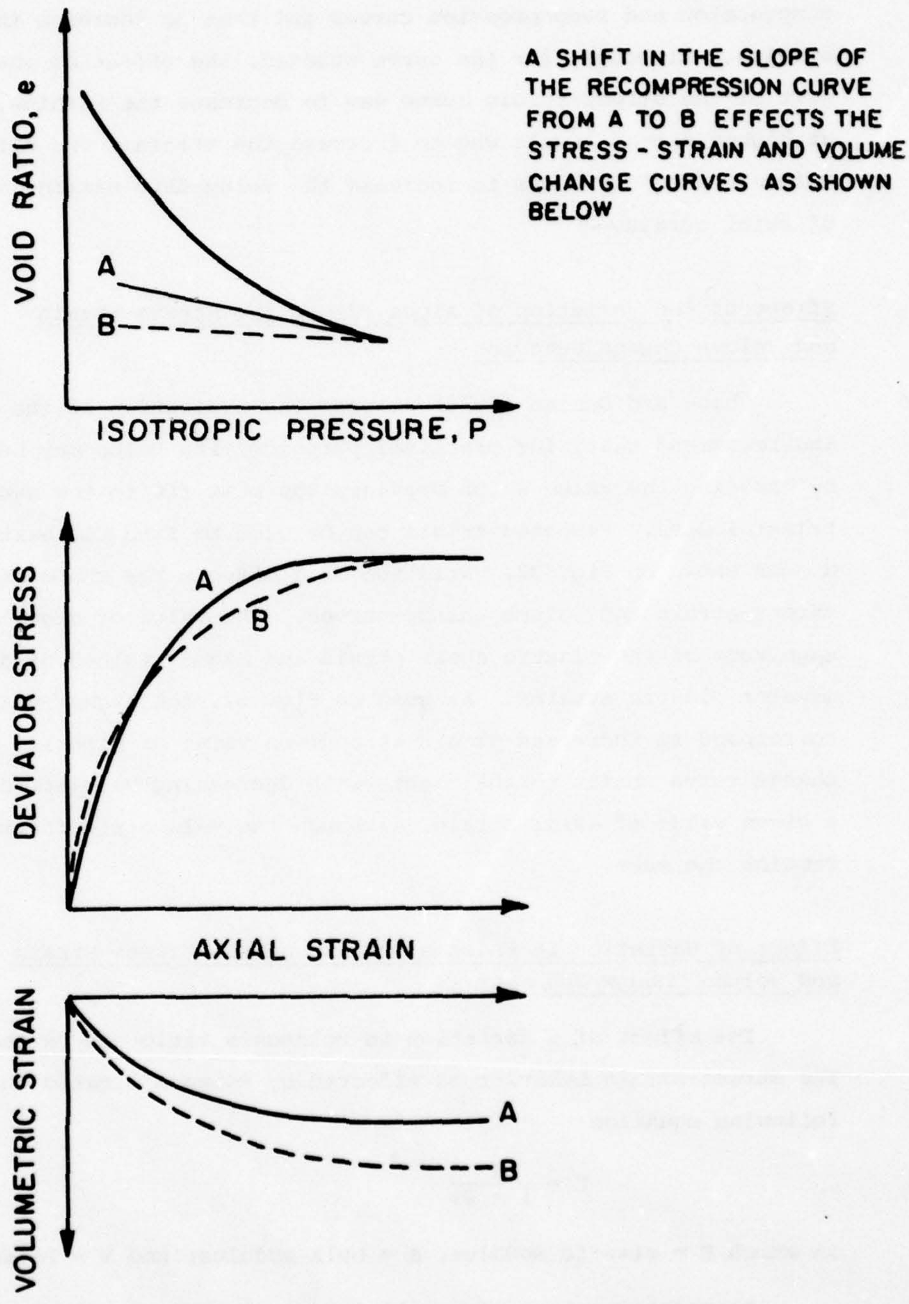


Fig. 31 EFFECT OF A SHIFT IN THE SLOPE OF THE RECOMPRESSION CURVE ON STRESS-STRAIN AND VOLUME CHANGE BEHAVIOR CALCULATED USING THE CAM CLAY MODEL

compression and recompression curves and thus to increase the plastic strains. Although, for the curve studied, the effect on the initial part of the stress-strain curve was to decrease the strains, the effect at higher stress levels was to increase the strain. The effect on the volume change curve was to increase the volumetric strains at all values of axial strain.

Effect of the Variation of Alpha (α) on the Stress-Strain and Volume Change Behavior

Chang and Duncan (1978) discuss the evaluation of the parameter α and recommend that, for practical purposes, its value may be determined by choosing the value which provides the best fit to the available triaxial data. Repeated trials can be used to find the best value of α. As shown on Fig. 32, variation of α effects the shape of both the stress-strain and volume change curves. The value of α governs the magnitude of the plastic shear strain and higher values of α result in greater plastic strains. As seen on Fig. 32, the higher values of α correspond to increased strain at a given value of stress. The volume change curve shifts to the right, with decreasing volumetric strain at a given value of axial strain, although the volumetric strain at failure remains the same.

Effect of Variation in Poisson's Ratio on the Stress-Strain and Volume Change Behavior

The effect of a variation in Poisson's ratio can be seen in Fig. 33. The stress-strain behavior is affected by Poisson's ratio through the following equation

$$E = \frac{B}{1 - 2\nu} \tag{12}$$

in which E = elastic modulus, B = bulk modulus, and ν = Poisson's ratio.

Decreasing values of ν will lead to increased values of E and a stiffer stress-strain curve as shown on Fig. 33. Lower volume changes are associated with increasing values of ν. For $\nu = 0.5$ no elastic volumetric strain would occur.

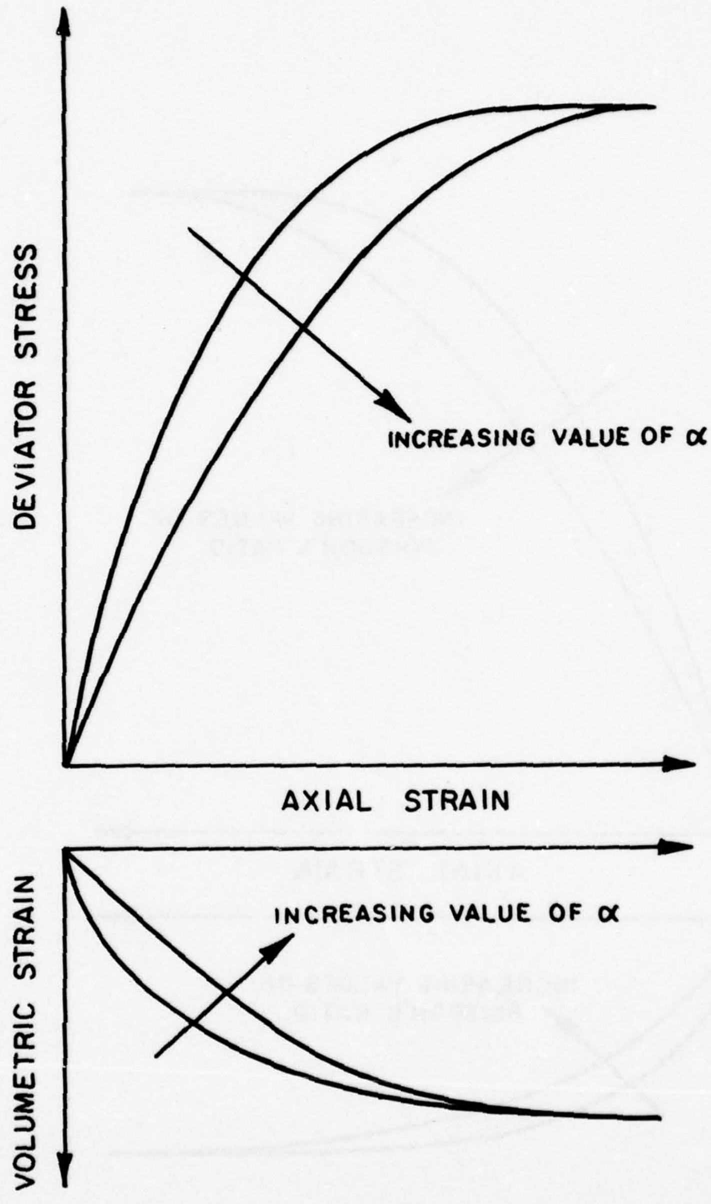


Fig. 32 EFFECT OF VARIATION OF ALPHA (α) ON THE STRESS-STRAIN AND VOLUME CHANGE BEHAVIOR FOR THE CAM CLAY MODEL

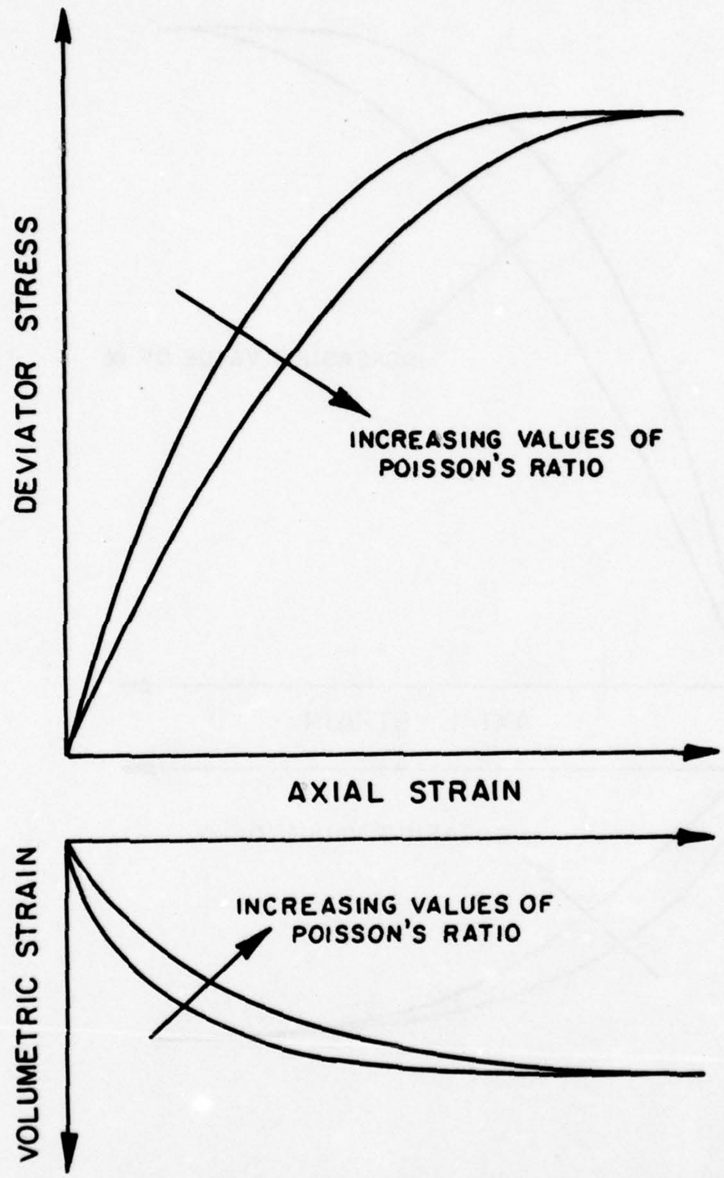


Fig. 33 EFFECT OF VARIATION IN POISSON'S RATIO ON THE STRESS-STRAIN AND VOLUME CHANGE BEHAVIOR FOR THE CAM CLAY MODEL

Calculated Stress-Strain and Volume Change Behavior

This portion of the study was focused on the ability of the Cam Clay model to calculate the stress-strain and volume change curves of soils with dilatant behavior. Three cohesionless soils were chosen, all of which exhibited dilatant behavior. For all the soils considered there was the problem of determining the volume change relationship for the isotropic consolidation tests because the measured volume changes all included the effects of membrane penetration. As previously discussed, this problem was handled by determining which e-p curve would provide the best fit to measured triaxial data. The e-p curve and the values of B, α and ν were all varied to provide the best fit achievable with a reasonable amount of effort. The data presented in the subsequent sections represent the adjusted curves.

Costa Rica Sandy Gravel

The e-p curve for the Costa Rica sandy gravel is shown on Fig. 28. The isotropic consolidation tests did not cover the full range of interest and it was therefore necessary to extrapolate the data to include the full stress range of interest. The measured and predicted stress-strain and volume change curves are shown on Fig. 34. The predicted stress-strain curves are in good agreement with the measured curves for all three tests. The predicted volume change curves are in good agreement for the low and intermediate confining pressures, and the predicted volumetric strains are about 20% to 30% too high for the highest confining pressure.

Modeled Oroville Dam Material

The e-p relationship and the stress-strain and volume change curves for the modeled Oroville material are shown on Figs. 35 and 36, respectively. The measured e-p curve had to be extrapolated to cover the stress range of interest. The predicted and measured stress-strain curves are in good agreement, even though the curves represent a very large range of confining pressures.

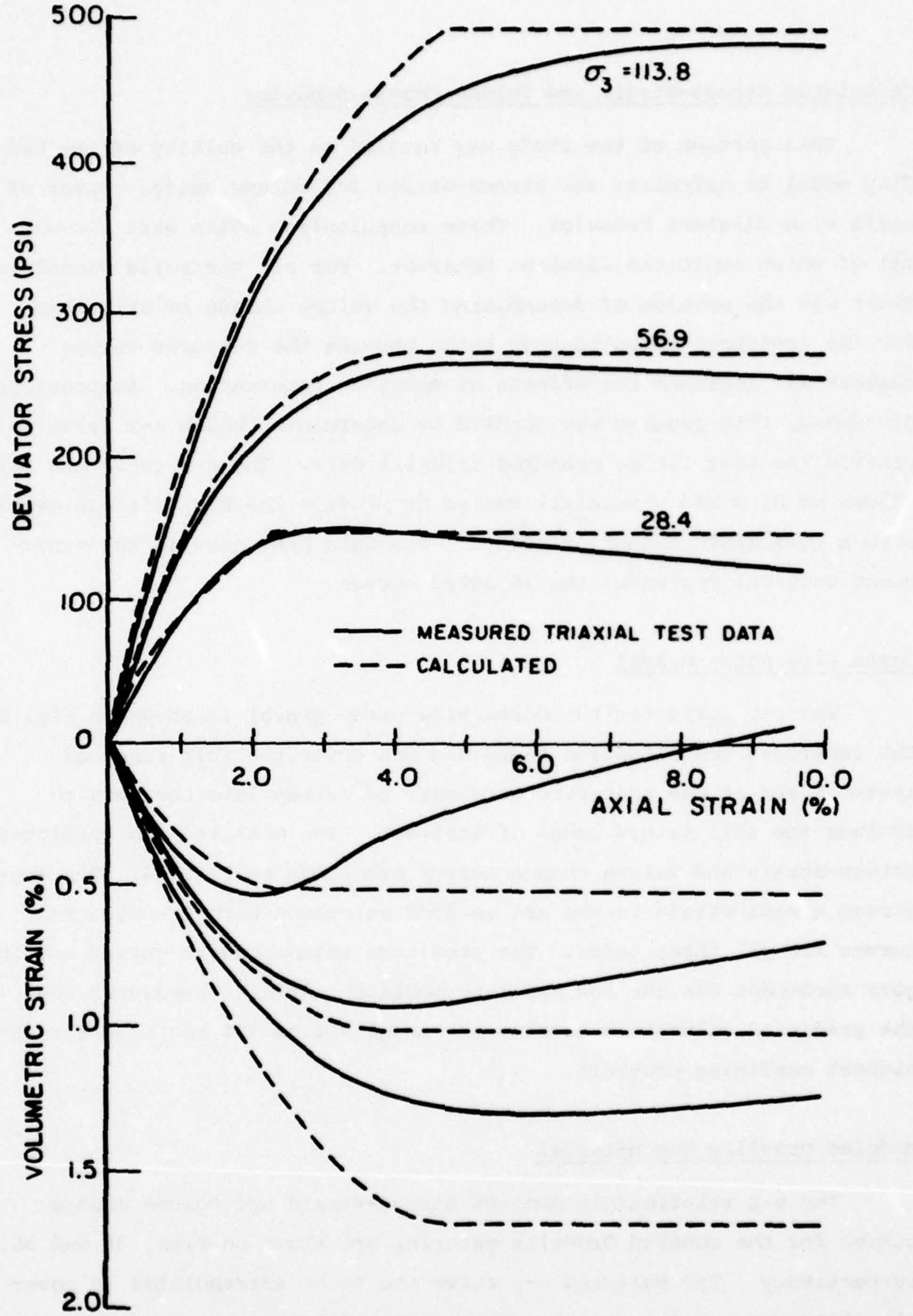


Fig. 34 COMPARISON OF MEASURED STRESS-STRAIN AND VOLUME CHANGE CURVES WITH THOSE CALCULATED USING ADJUSTED PARAMETERS FOR THE CAM CLAY MODEL, FOR THE COSTA RICA SANDY GRAVEL

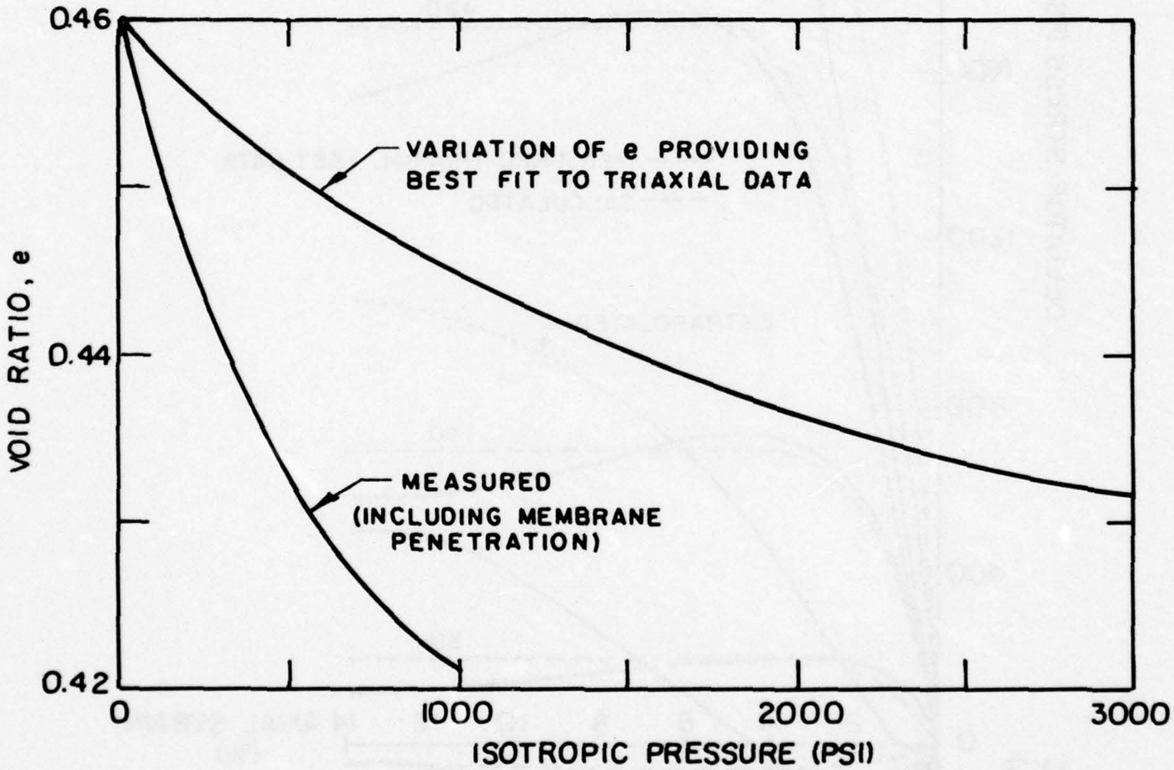


Fig. 35 VARIATION OF VOID RATIO WITH ISOTROPIC PRESSURE FOR THE MODELED OROVILLE MATERIAL

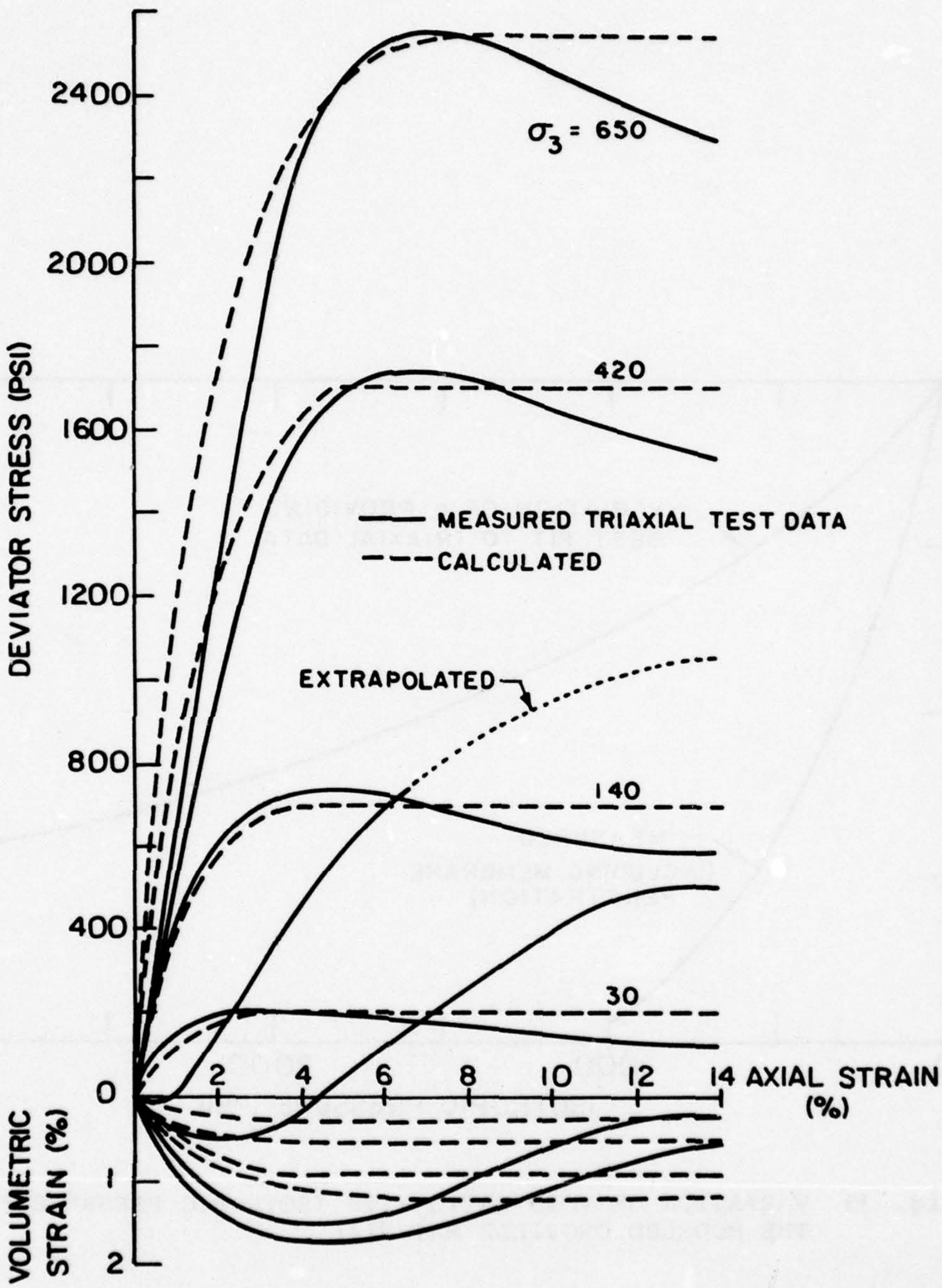


Fig. 36 COMPARISON OF MEASURED STRESS-STRAIN AND VOLUME CHANGE CURVES WITH THOSE CALCULATED USING ADJUSTED PARAMETERS FOR THE CAM CLAY MODEL, FOR THE MODELED OROVILLE MATERIAL

The modeled Oroville material is very dilative and the calculated volumetric strains are not in good agreement with those measured. At the lowest confining pressure the calculated and measured curves differ greatly. At the higher confining pressures the measured volumetric strains are 50% to 60% greater than those calculated.

Monterey Sand

The e-p relationship and the calculated and measured stress-strain and volume change curves are shown on Figs. 37 and 38. As with the other soils it was necessary to extrapolate the data from the e-p curve to cover the full stress range of interest. The calculated and the measured stress-strain curves are in good agreement. It can be seen that the calculated deviator stress at failure is too high for the intermediate confining pressure and too low for high confining pressure. The measured variation of ϕ with log (σ_3/p_a) is not linear for these three tests, and the straight line which was fitted to the three points produces the failure stresses shown on Fig. 38.

The Monterey sand is a very dilatant material and the calculated volumetric strains are not in good agreement with those measured. For all confining pressures shown the calculated and measured volumetric strains differ greatly at axial strains as low as 1% to 2%.

Summary of the Characteristics of the Cam Clay Model

The Cam Clay model provides good agreement between calculated and measured triaxial stress-strain data for stresses up to the peak stress. The model does not accurately predict stresses and strains at strains past the peak stress, as shown by the calculated and measured stress-strain curves for the modeled Oroville material at the highest confining pressure.

The model is not able to represent dilatant volumetric strains.

The parameters required for the Cam Clay model can be determined from the results of:

1. Isotropic triaxial consolidation tests, and

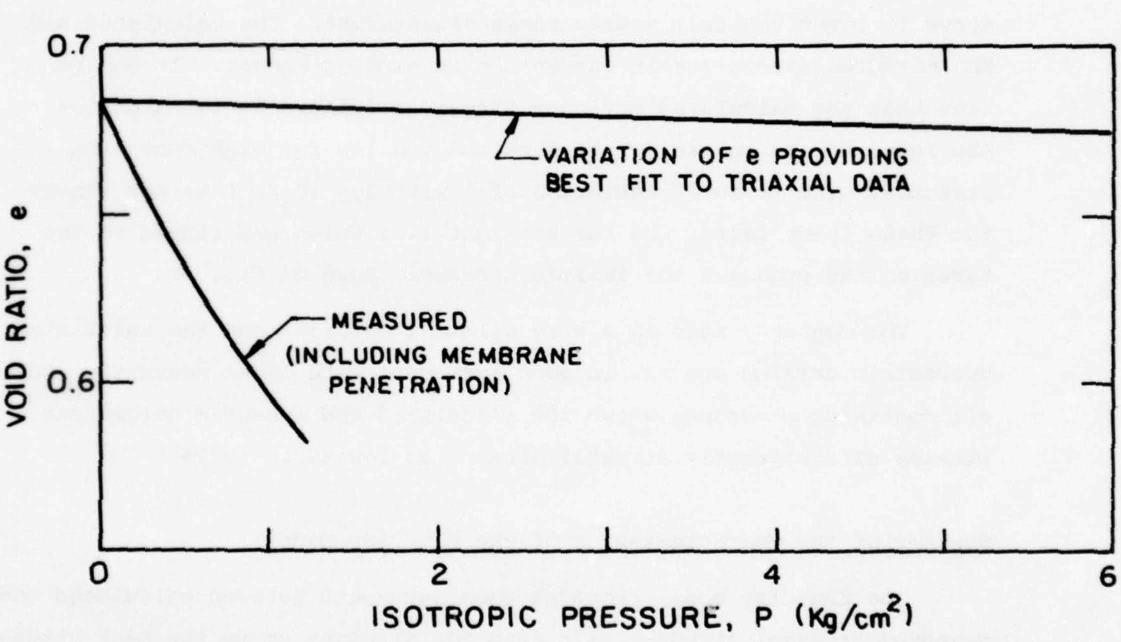


Fig. 37 VARIATION OF VOID RATIO, e, WITH ISOTROPIC PRESSURE, P, FOR MONTEREY SAND

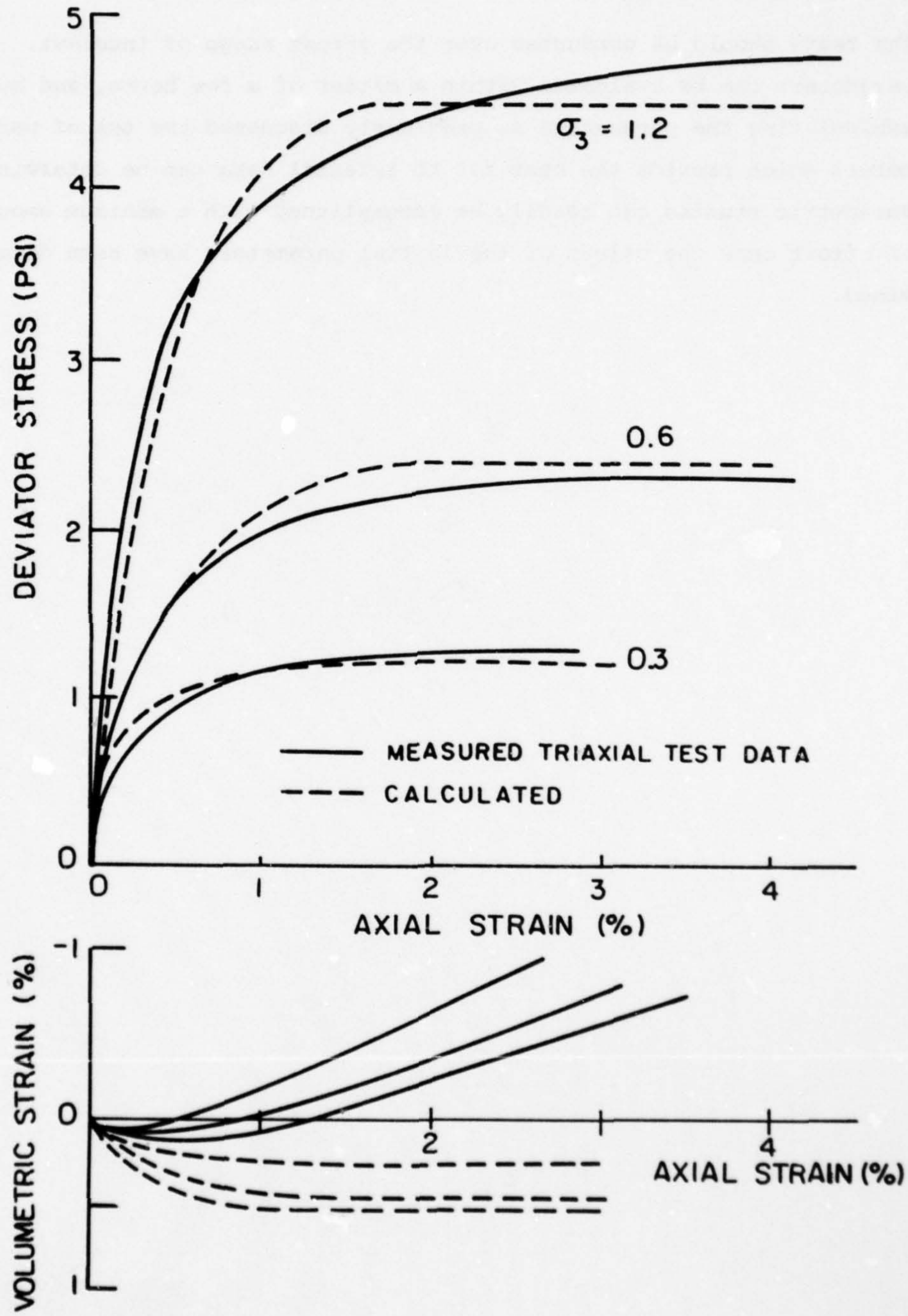


Fig. 38 COMPARISON OF MEASURED STRESS-STRAIN AND VOLUME CHANGE CURVES, WITH THOSE CALCULATED USING ADJUSTED PARAMETERS FOR THE CAM CLAY MODEL, FOR MONTEREY SAND

2. Conventional triaxial compression tests.

The tests should be conducted over the stress range of interest. The parameters can be evaluated within a matter of a few hours, and by manipulating the parameters as previously discussed the set of parameters which provide the best fit to triaxial data can be determined. Parametric studies can readily be accomplished with a minimum amount of effort once the values of the initial parameters have been determined.

HYPERBOLIC MODEL

The hyperbolic stress-strain relationships were developed for use in nonlinear incremental analyses of soil deformations. The hyperbolic relationships were developed by Duncan and Chang (1970) and by Duncan et al (1978). The latter reference includes the latest modification to the model. In the latest version of the hyperbolic model, the relationships between strain increments and the corresponding stress increments are expressed in terms of the parameters E (Young's modulus) and B (bulk modulus).

The hyperbolic relationships account for three important characteristics of the stress-strain behavior of soils, namely nonlinearity, stress dependency and inelasticity. In the following sections, the procedures used to develop the parameters are illustrated for the Costa Rica sandy gravel. The procedures used have been outlined by Duncan et al (1978). The calculated and measured stress-strain and volume change curves are shown in subsequent sections for the Costa Rica sandy gravel, the modelled Oroville Dam material, and Monterey No. 0 sand.

Development of Hyperbolic Parameters

The development of the hyperbolic parameters are illustrated using the adjusted triaxial stress-strain and volume change curves for the Costa Rica sandy gravel. The curves were adjusted as discussed in the section on the Al-Shawaf - Powell model to remove inconsistencies in the behavior. The adjusted curves are shown on Fig. 39.

Evaluation of ϕ_o and $\Delta\phi$

A discussion on the evaluation of ϕ and $\Delta\phi$ was presented in the section on the Cam Clay model. The variation of ϕ with confining pressure is shown on Fig. 40. The value of ϕ_o is equal to the value of ϕ for σ_3 equal to one atmosphere, and the value of $\Delta\phi$ is the variation in ϕ for one log cycle of confining pressure.

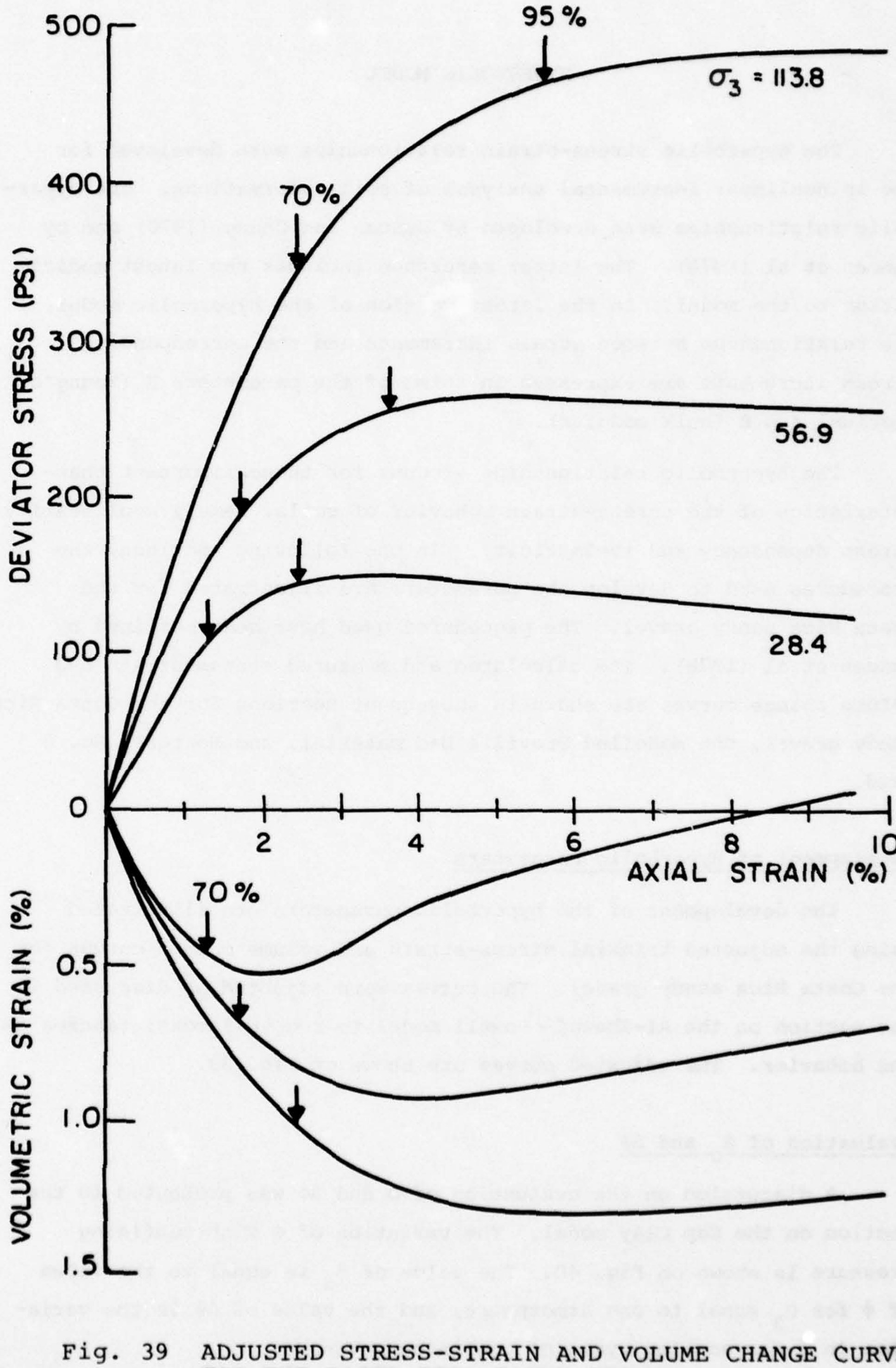


Fig. 39 ADJUSTED STRESS-STRAIN AND VOLUME CHANGE CURVES FOR THE COSTA RICA SANDY GRAVEL

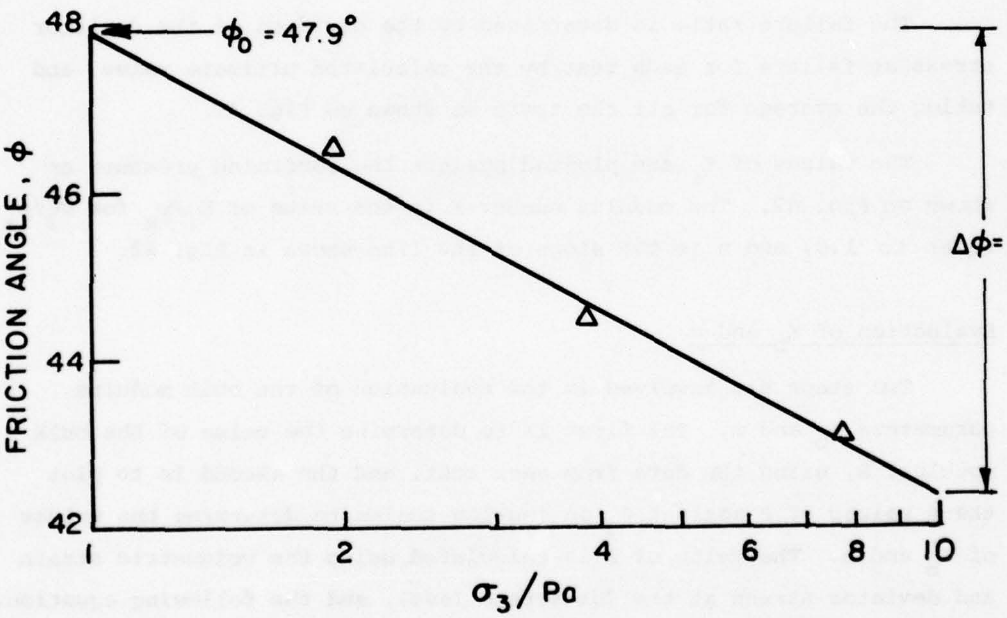


Fig. 40 VARIATION OF FRICTION ANGLE WITH CONFINING PRESSURE FOR THE COSTA RICA SANDY GRAVEL

Evaluation of K and n

Two steps are involved in evaluating the modulus parameters K and n. The first is to determine the initial Young's modulus, E_i, for each test and second to plot E_i against σ_3 on log-log scales to determine the values of K and n. First the values of axial strain and deviator stress at stress levels of 70% and 95% were determined from Fig. 39 and plotted in the transformed stress-strain plot as shown on Fig. 41. The reciprocal of the intersection of the curves at zero axial strain is the initial value of E, and the reciprocal of the slope of the curves is the ultimate value of deviator stress for the hyperbolic curve.

The failure ratio is determined by the dividing of the deviator stress at failure for each test by the calculated ultimate value, and taking the average for all the tests as shown on Fig. 41.

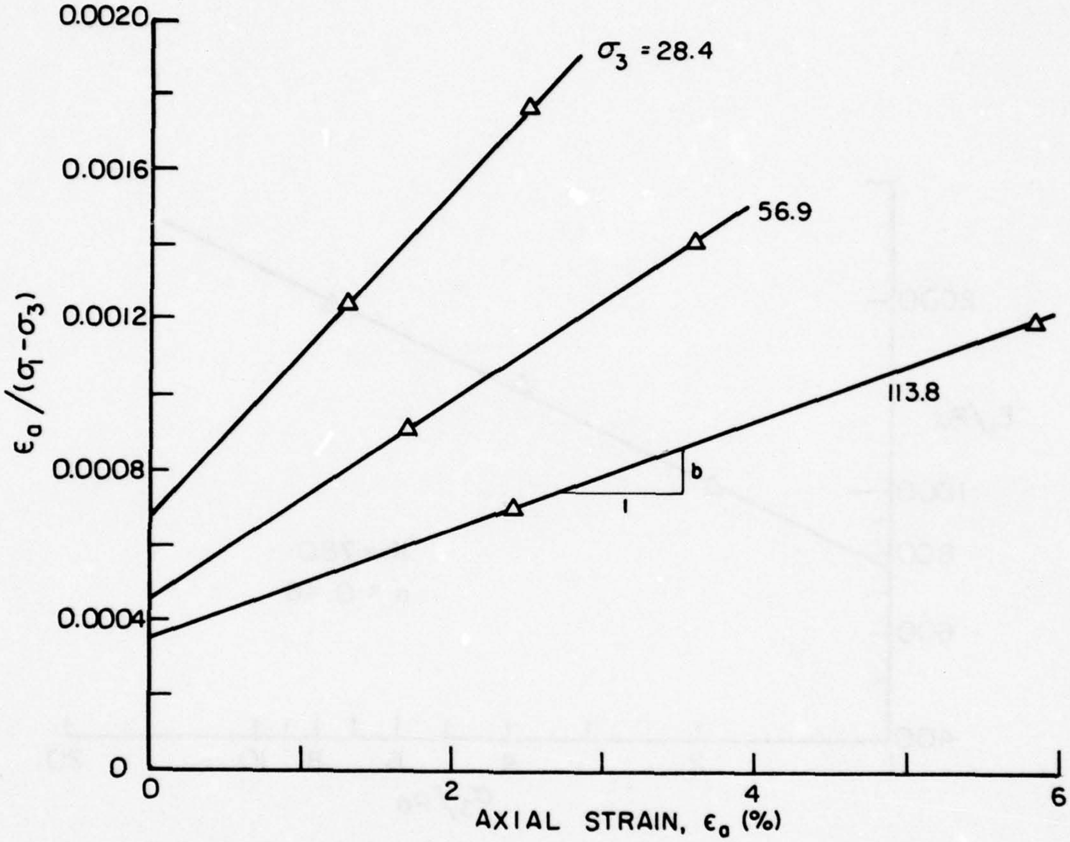
The values of E_i are plotted against the confining pressure as shown on Fig. 42. The modulus number K is the value of E_i/p_a for σ_3/p_a equal to 1.0, and n is the slope of the line shown in Fig. 42.

Evaluation of K_b and m

Two steps are involved in the evaluation of the bulk modulus parameters K_b and m. The first is to determine the value of the bulk modulus, B, using the data from each test, and the second is to plot these values of B against σ_3 on log-log scales to determine the values of K_b and m. The value of B is calculated using the volumetric strain and deviator stress at the 70% stress level, and the following equation.

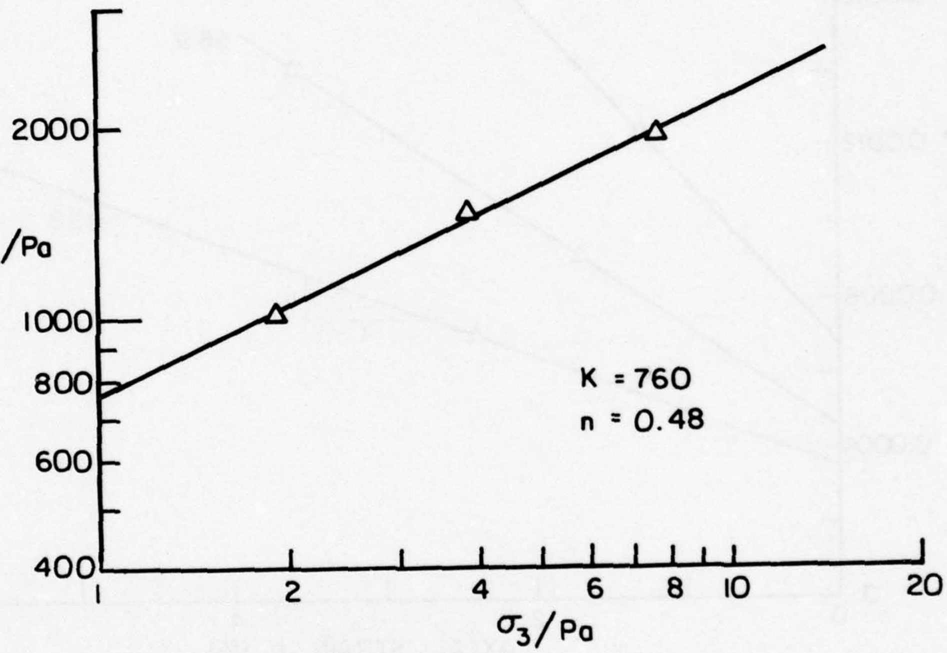
$$B = \frac{(\sigma_1 - \sigma_3)}{3\varepsilon_v} \qquad (13)$$

The values of B/p_a are plotted against σ_3/p_a as shown on Fig. 43. The value of B/p_a at σ_3/p_a equal to 1.0 is the bulk modulus number K_b, and the slope of the line is the value of m.



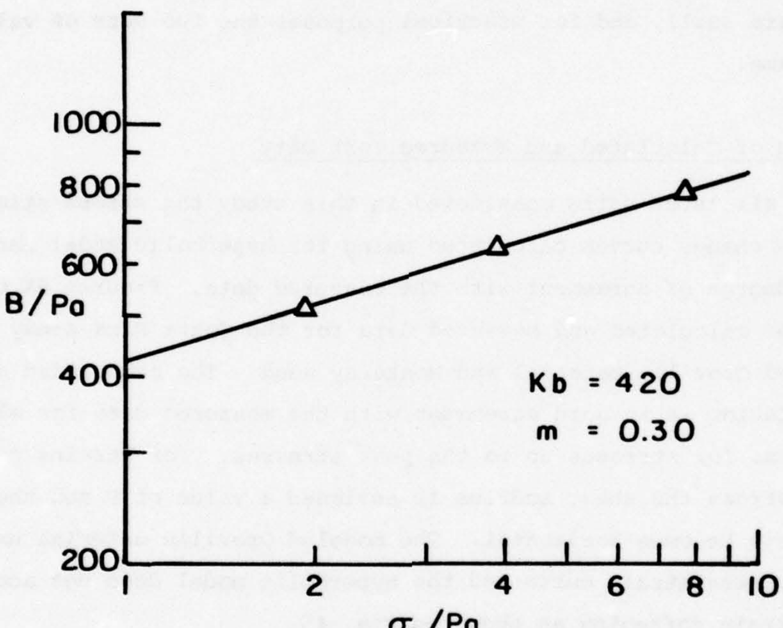
σ_3	a	$E_i = 1/a$	b	$(\sigma_1 - \sigma_3)_{ult}$	$(\sigma_1 - \sigma_3)_f$	R_f
28.4	0.000067	14 925	0.00433	231	151	0.65
56.9	0.000046	21 740	0.00268	373	268	0.72
113.8	0.000035	28 570	0.00156	641	491	0.77
						0.71 = AVERAGE R_f

Fig. 41 TRANSFORMED STRESS-STRAIN PLOT FOR THE COSTA RICA SANDY GRAVEL



σ_3	σ_3/Pa	E_i	E_i/Pa
28.4	1.93	14925	1015
56.9	3.87	21740	1480
113.8	7.74	28570	1940

Fig. 42 VARIATION OF THE INITIAL TANGENT MODULUS WITH CONFINING PRESSURE FOR THE COSTA RICA SANDY GRAVEL



σ_3	σ_3/Pa	B	B/Pa	$B = \frac{(\sigma_1 - \sigma_3)}{3\,\epsilon_v}$ at
28.4	1.93	7500	510	
56.9	3.87	9200	630	70% STRESS LEVEL
113.8	7.74	11500	780	

Fig. 43 VARIATION OF BULK MODULUS WITH CONFINING PRESSURE FOR THE COSTA RICA SANDY GRAVEL

Computer Program for Determining Parameter Values

A computer program has been developed for determination of the stress-strain and volume change parameters using least square procedures for fitting the curves illustrated in Figs. 42 and 43. A users' guide and listing are presented in the report by Duncan et al (1978).

Parameter values computed using the computer program are compared to those calculated by hand for the Costa Rica sandy gravel in Table 2. It may be seen that although the values are not identical the differences are small, and for practical purposes the two sets of values are the same.

Comparison of Calculated and Measured Test Data

For all three soils considered in this study the stress-strain and volume change curves calculated using the hyperbolic model had about the same degree of agreement with the measured data. Figures 44 to 46 compare the calculated and measured data for the Costa Rica sandy gravel, the modeled Oroville material and Monterey sand. The calculated stress-strain behavior is in good agreement with the measured data for all three soils, for stresses up to the peak stresses. For strains past the peak stress the shear modulus is assigned a value of 0 and the stress-strain curve becomes horizontal. The modeled Oroville material exhibits a peaked stress-strain curve and the hyperbolic model does not account for the strain softening as shown on Fig. 45.

The hyperbolic relationships do not model dilatant volume changes resulting from shear stresses, and thus always indicate compression under increasing values of stress, even though the test data may indicate dilation at larger values of axial strain. The soils considered in this study all exhibit dilation to some degree. For all three soils the comparison of measured and calculated volume changes are in good agreement for volume changes up to the maximum compressive volumetric strain measured.

Table 2. Comparison of Stress-Strain Parameter Values for Costa Rica Sandy Gravel Determined using Computer Program and by Hand Calculation

Parameter	Computer Program Value	Hand Calculation Value
K	717	760
n	.53	.48
ϕ	48.2	47.9
$\Delta\phi$	5.8	5.5
R_f	.71	.71
K_b	416	420
m	.31	.30

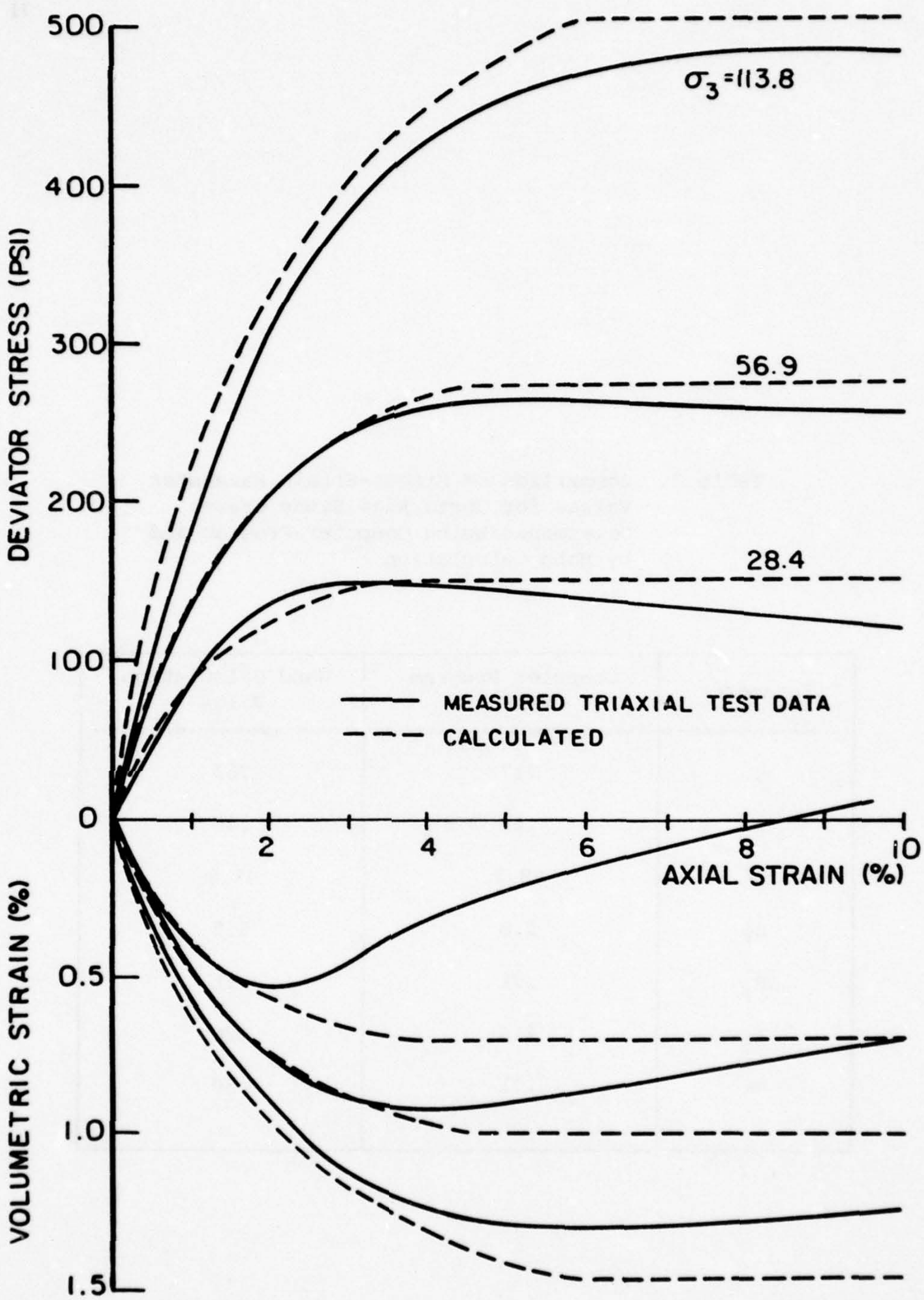


Fig. 44 COMPARISON OF MEASURED STRESS-STRAIN AND VOLUME CHANGE CURVES, WITH THOSE CALCULATED USING THE HYPERBOLIC MODEL, FOR COSTA RICA SANDY GRAVEL

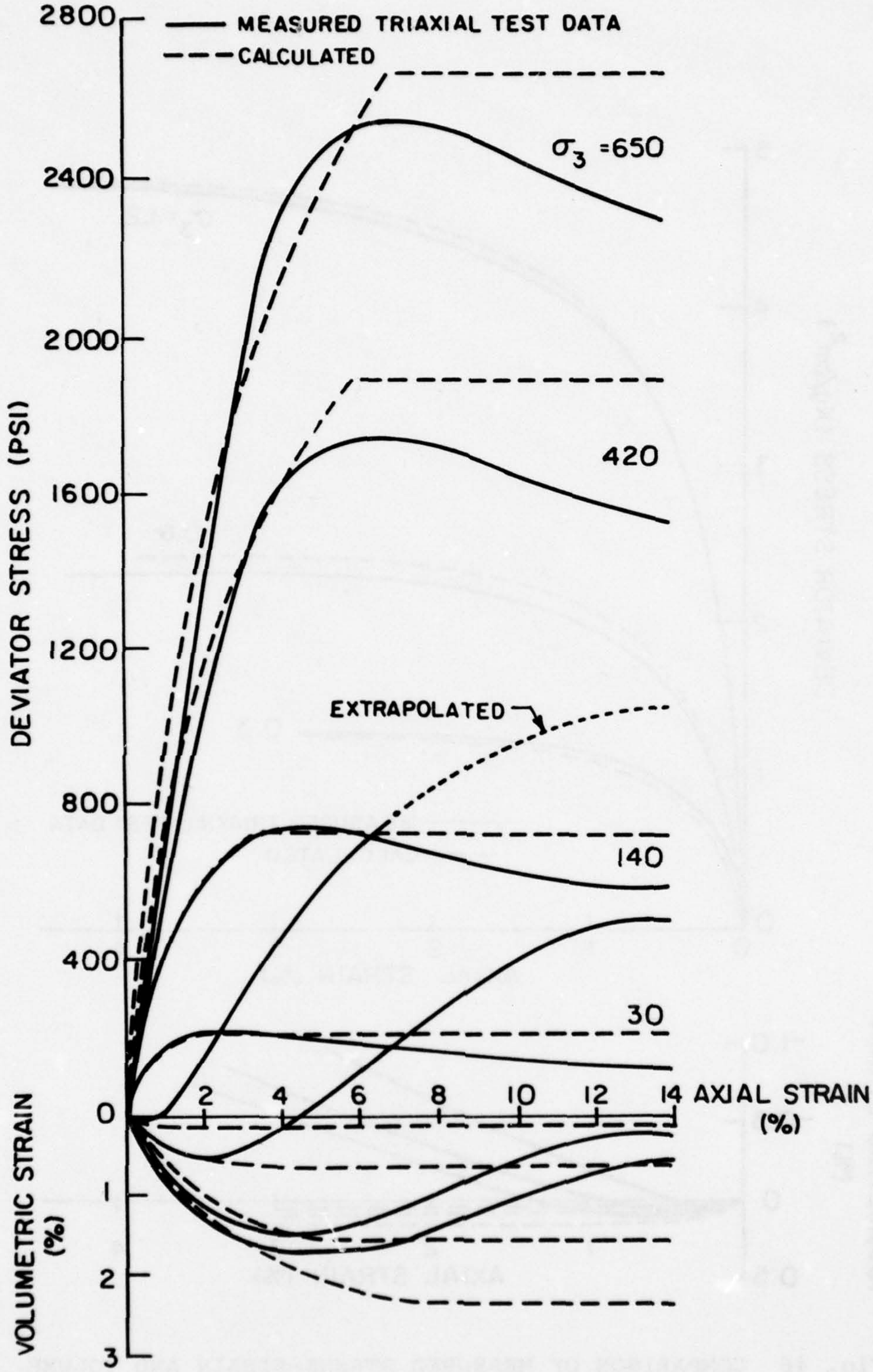


Fig. 45 COMPARISON OF MEASURED STRESS-STRAIN AND VOLUME CHANGE CURVES WITH THOSE CALCULATED USING THE HYPERBOLIC MODEL, FOR THE MODELED OROVILLE MATERIAL

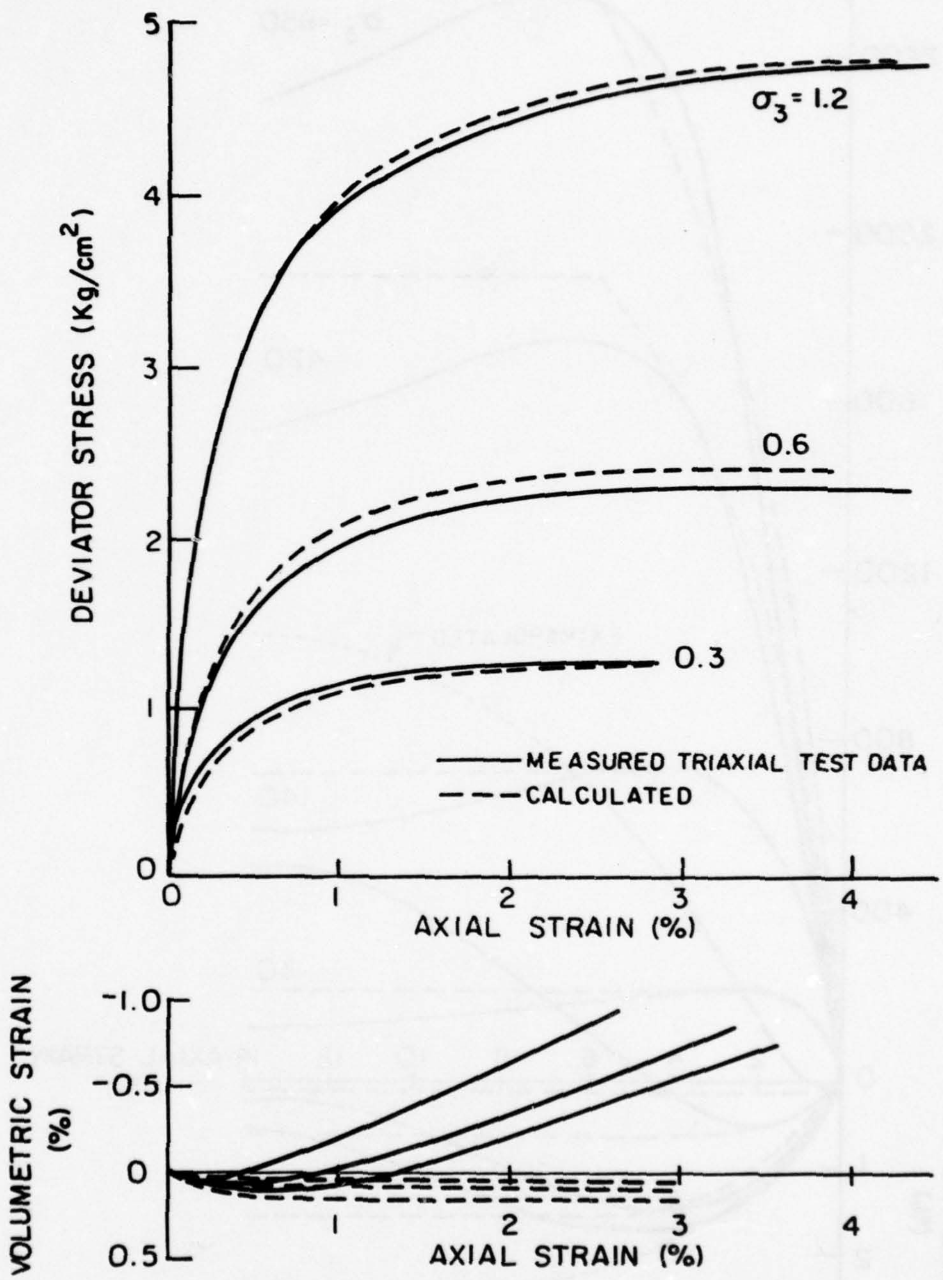


Fig. 46 COMPARISON OF MEASURED STRESS-STRAIN AND VOLUME CHANGE CURVES WITH THOSE CALCULATED USING THE HYPERBOLIC MODEL, FOR MONTEREY SAND

Summary of Hyperbolic Model

As can be seen, the hyperbolic model is capable of representing triaxial data with good agreement for stresses up to the peak, and volumetric strains up to the maximum compressive volumetric strains even for the dilative materials used in this study. The hyperbolic model can thus provide a good representation of soil stress-strain behavior for stresses up to failure. The hyperbolic model will not provide an accurate representation of soil stress-strain behavior for stresses past the peak stresses in soils which exhibit dilative behavior.

The model is extremely easy to use in that the parameters are developed from triaxial compression tests and can be calculated using the computer program presented by Duncan et al (1978). Values of the parameters have been calculated for many soils and the data available can be used to judge the reliability of parameter values determined from laboratory test results.

Summary of the Characteristics of the Al-Shawaf - Powell, the Cam Clay, and the Hyperbolic Models

The characteristics of the three soil models were studied by reproducing the triaxial test data from which the parameters were developed. The soils used in this study all exhibited dilative behavior for the range of pressures for which the tests were conducted.

As previously discussed the distinguishing characteristic of the Al-Shawaf - Powell model is its ability to model the dilation effects on the stress-strain and volume change behavior of soils. The Cam Clay and Hyperbolic models do not include effects of shear dilation.

In discussing the ability of the models to reproduce triaxial test data the behavior will be separated into stresses up to the peak stress and stresses past the peak stress. Similarly, volume change behavior will be considered in two parts: volumetric strains up to the maximum compressive volumetric strain, and volumetric strains past the maximum. The characteristics for the three models are summarized in Table 3. All three models are capable of modeling the stress-strain behavior of triaxial tests for stresses up to the peak stress, with very good agreement with measured data. Thus there is very little to distinguish between the three models in their ability to reproduce triaxial stress-strain behavior at stresses less than the ultimate stresses. At stresses past the peak stress, where the soil exhibits the effect of dilation with a peaked stress-strain curve, only the Al-Shawaf - Powell model can account for the strain softening effects. The hyperbolic and Cam Clay models both have a horizontal stress-strain curve at strains larger than that corresponding to the ultimate stress. The differences between the calculated and measured data increase with the increasing effects of dilation at large strains for the Cam Clay and hyperbolic models. The modeled Oroville material is a good example of this behavior and is shown on Figs. 36 and 45.

The Al-Shawaf - Powell and hyperbolic models model volumetric strains up to the peak compressive volumetric strain quite accurately. For the dilative soils studied it was very difficult to match the measured data with the Cam Clay model. The Al-Shawaf - Powell model is

Table 3. Summary of the Observations Made Concerning The Characteristics of the Al-Shawaf - Powell, Cam Clay and Hyperbolic Models for the Three Dilative Soils Studied

	Al-Shawaf - Powell	Cam Clay	Hyperbolic
Models stress-strain behavior up to peak stresses	Yes	Yes	Yes
Models stress-strain behavior past peak stresses	Yes	No	No
Models volumetric strains up to the maximum compressive volumetric strain	Yes	difficult to model	Yes
Model dilative volumetric behavior	Yes	No	No
Time required to develop parameters	3-5 days	2-3 hours	1 hour
Time required to adjust parameters to develop a good fit to triaxial data	3-4 days	1 day	0
Laboratory tests required to develop parameters	Triaxial Compression Triaxial Consolidation Triaxial Extension	Triaxial Compression Triaxial Consolidation	Triaxial Compression
Computational Effective Computer Time for a single element with 2 degrees of freedom	21 sec	9 sec	6 sec

the only model capable of modeling dilative volumetric behavior. The hyperbolic and Cam Clay model both have horizontal volumetric strain curves for strains larger than that required to reach the maximum compressive volumetric strain.

Development of Parameters

The Al-Shawaf - Powell model requires the most extensive testing program to obtain the data required to develop the parameters. The parameters can be obtained from the following tests:

1. Triaxial Compression
2. Triaxial Extension
3. Isotropic Triaxial Consolidation

Once the data from the tests are available 3 to 5 days are required to evaluate the required parameters. Developing the parameters often requires judgment, and the calculated parameters may not provide good agreement between calculated and measured test data without adjustment. It is thus necessary to change the parameters for the paths which triaxial tests follow in the state space. Because these paths will not include every zone in the state space, it will be necessary to ensure that the parameters in the zones between those which have been adjusted still vary consistently from one zone to the next. This process may take another 3 to 4 days of work. Therefore it may take as long as one to two weeks to develop parameters which provide good agreement between measured and calculated data for all values of stress.

If it is desired to examine the effect of a range of stress-strain data by examining upper and lower bounds of behavior, the process would be very time consuming for the Al-Shawaf - Powell model. The values of the parameters are not related to soil type alone; they also depend on the sizes of the I and J zones employed.

The parameters for the Cam Clay model can be developed from the following tests:

1. Triaxial Compression
2. Isotropic Triaxial Consolidation.

Once the data from the tests are available the parameters can be developed within 2 to 3 hours. Adjustment of the parameters to provide the best fit to measured triaxial test data will require about 1 day. Therefore parametric studies can be conducted more readily because the effort required to develop the parameters is smaller.

The hyperbolic parameters can be developed from triaxial compression tests alone, and using the computer program SP-5 the parameters can be developed within about one hour. The parameters are developed using least square procedure to provide the best fit curves. No further work is necessary to modify the parameters to provide the best fit to triaxial data.

A comparison of the effective computer time for the three models is shown on Table 3. All of the computer times listed are for a single element system with 2 degrees of freedom. The values shown can be considered as a rough guide as to the relative computer costs associated with each model.

Summary

The purpose of this study was to evaluate the characteristics of the Al-Shawaf - Powell for use in practical problems. This was accomplished by comparing the characteristics of the Al-Shawaf - Powell model with the Cam Clay and hyperbolic models.

For most practical problems involving the design of earth structures such as dams, the stress level is usually in the range of 50% to 80% of the ultimate stress, and most finite element analyses are concerned with stresses and deformations of soil masses at stresses less than failure. The hyperbolic and Al-Shawaf - Powell model have both been shown to accurately model stress-strain and volume change behavior at stress less than the peak stresses. The Cam Clay model can reproduce the stress-strain behavior with good agreement, but the model cannot reproduce the volumetric behavior of dilatant soils accurately. For cases where the behavior at or past the failure

stresses are of interest, the Al-Shawaf - Powell model may be useful, although data is not available for multi-element systems at this time.

The time required to develop the parameters for the Al-Shawaf - Powell model is considerable when compared with the hyperbolic model. Because numerical techniques are used in the Al-Shawaf - Powell models, it is not possible to develop typical parameters for various types of soil as has been done for the hyperbolic model.

References

1. Al-Shawaf, T. and Powell, G. H. (1977) "Variable Modulus Model for Nonlinear Analysis of Soils," Proceeding of Symposium on Application of Computer Methods in Engineering, University of Southern California, Vol. 1, pp. 423-433.

2. Chang, C. S. and Duncan, J. M. (1977) "Analysis of Consolidation of Earth and Rockfill Dams," Report NO. TE 77-3, University of California, Berkeley, September 1977.

3. Duncan, J. M., Byrne, P., Wong, K. S. and Mabry, P. (1978) "Strength, Stress-Strain and Bulk Modulus Parameters for Finite Element Analyses of Stresses and Movements in Soil Masses," Report No. UCB/GT/78-02, University of California, Berkeley, April 1978.

4. Duncan, J. M. and Chang, C-Y. (1970) "Nonlinear Analysis of Stresses and Strain in Soils," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 96, No. SM5, September 1970.

5. Lee, L. L. (1965) "Triaxial Compressive Strength of Saturated Sand Under Seismic Loading Conditions," Dissertation Presented to the University of California, Berkeley, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

6. Marachi, D. N., Chan, C. K., Seed, H. B. and Duncan, J. M. (1969) "Strength and Deformation Characteristics of Rockfill Materials," Report No. 69-5, University of California, Berkeley, September 1969.

7. Mondkar, D. P. and Powell, G. H. (1975) "Static and Dynamic Analysis of Nonlinear Structures," Report No. EERC 75-10, Earthquake Engineering Research Center, University of California, Berkeley, 1975.

8. Mondkar, D. P. and Powell, G. H. (1975) "ANSR-I General Purpose Program for Analysis of Nonlinear Structural Response," Report No. EERC 75-37, Earthquake Engineering Research Center, University of California, Berkeley, 1975.

9. Wong, K. S. (1978) "Elasto-Plastic Finite Element Analyses of Passive Pressure Tests," Dissertation presented to the University of California, Berkeley, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

10. Wong, K. S. and Duncan, J. M. 91974) "Hyperbolic Stress-Strain Parameters for Nonlinear Finite Element Analyses of Stresses and Movements in Soil Masses," Report No. TE 74-3, University of California, Berkeley, July, 1974.

APPENDIX

VARIABLE MODULUS MODEL FOR NONLINEAR ANALYSIS OF SOILS

Taha Al-Shawaf (I) and Graham H. Powell (II)

Summary

A variable modulus modelling procedure for nonlinear analysis of soils is presented. The procedure accounts for stiffening behavior under volumetric strain, softening behavior under distortional strain, the influence of hydrostatic stress on shear strength, and the shear dilation effect. A numerical modelling process based on piecewise linear behavior (piecewise constant moduli) is used, without the use of analytical expressions for material properties. The modulus values are determined from the results of hydrostatic and triaxial tests.

The theoretical basis of the method is presented, the procedure for calculating the tangent moduli is described, and an example is presented to demonstrate that test results can be reproduced.

1. Introduction

Soils exhibit many complexities in their constitutive relationships, the most important of which are as follows.

(1) Progressive stiffening behavior under volumetric strain.

(2) Progressive softening behavior under distortional strain. For frictional materials a peak distortional strength may be reached with strength loss for subsequent increases in strain.

(3) Distortional strengths and stiffnesses which are strongly dependent on the hydrostatic stress.

(4) For frictional materials, substantial coupling between distortional and volumetric strain (shear dilation).

A comprehensive review of available constitutive models for soil and rock materials, and their application in finite element solutions, has been prepared by Desai [1]. Few of the available models account for all of the above listed complexities, and none of them appears to be sufficiently general for widespread practical application.

The modelling procedure described herein is of variable tangent modulus type. It is similar in principle to the procedure described by Nelson and Baron [2,3] but differs in two important respects, as follows.

(1) The model accounts for shear dilation.

I. Department of Civil Engineering, University of California, Berkeley.
II. Department of Civil Engineering, University of California, Berkeley.

(2) The tangent moduli are obtained not from analytical expressions, but by means of a numerical procedure which selects piecewise constant values of the moduli so that the behavior observed in hydrostatic and triaxial tests is reproduced.

2. Basic Relationships

Because the shear dilation effect may have a large influence on the behavior of confined soils, it is important that this effect be modelled accurately. It is assumed that any increment of volumetric strain can be uncoupled into (a) an increment associated with change of hydrostatic stress and (b) an increment associated with change of distortional strain. This type of assumption is common, having been made by Lomize et al [4], Vyalov [5], Bazant et al [6] and others.

In terms of mean strain the assumption is

$$\dot{\varepsilon}_m = \dot{\varepsilon}_m^h + \dot{\varepsilon}_m^d \tag{1}$$

in which: $\dot{\varepsilon}_m$ = mean strain rate; and superscripts h and d indicate the hydrostatic and distortional effects, respectively. The hydrostatic component is given by

$$\dot{\varepsilon}_m^h = \frac{\dot{\sigma}_m}{3K} \tag{2}$$

in which: $\dot{\sigma}_m$ = mean stress rate; and K = tangent value of bulk modulus. The distortional component is given by

$$\dot{\varepsilon}_m^d = D\,\dot{e}_I \tag{3}$$

in which: D = tangent value of dilatancy ratio;

$\dot{e}_I = \sqrt{0.5\,\dot{e}_{ij}\,\dot{e}_{ij}}$ = intensity of deviatoric strain rates,

$\dot{e}_{ij} = \dot{\varepsilon}_{ij} - \delta_{ij}\,\dot{\varepsilon}_m$ = deviatoric strain rate.

$\dot{e}_{ij}$ = strain rate; and δ_{ij} = Kronecker delta.

The deviatoric strain rate is given by

$$\dot{e}_{ij} = \frac{\dot{S}_{ij}}{2G} \tag{4}$$

in which $\dot{S}_{ij} = \dot{\sigma}_{ij} - \delta_{ij}\dot{\sigma}_m$ is the deviatoric stress rate; and G = tangent value of shear modulus.

3. Tangent Constitutive Matrix

The tangent constitutive relationship is derived from

$$\dot{\sigma}_{ij} = \delta_{ij}\,\dot{\sigma}_m + \dot{S}_{ij} \tag{5}$$

by making use of Eqs. 1, 2, 3 and 4 to give

$$\dot{\sigma}_{ij} = \delta_{ij}(3K\,\dot{\varepsilon}_m - 3KD\dot{e}_I) + 2G\dot{e}_{ij} \tag{6}$$

In matrix form, Eq. (6) can be written as

$$\begin{Bmatrix} \dot{\sigma}_{11} \\ \dot{\sigma}_{22} \\ \dot{\sigma}_{33} \\ \dot{\sigma}_{12} \\ \dot{\sigma}_{23} \\ \dot{\sigma}_{31} \end{Bmatrix} = \begin{bmatrix} E_1 & E_2 & E_2 & \cdot & \cdot & \cdot \\ E_2 & E_1 & E_2 & \cdot & \cdot & \cdot \\ E_2 & E_2 & E_1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 2G & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 2G & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 2G \end{bmatrix} \begin{Bmatrix} \dot{\varepsilon}_{11} \\ \dot{\varepsilon}_{22} \\ \dot{\varepsilon}_{33} \\ \dot{\varepsilon}_{12} \\ \dot{\varepsilon}_{23} \\ \dot{\varepsilon}_{31} \end{Bmatrix} - \begin{bmatrix} 3KD \\ 3KD \\ 3KD \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \dot{e}_I \quad (7)$$

in which $E_1 = K + 4G/3$ and $E_2 = K - 2G/3$; or, symbolically

$$\{\dot{\sigma}\} = [S]\,\{\dot{\varepsilon}\} - [T]\dot{\varepsilon}_I \quad (8)$$

If radial straining is assumed ($\dot{e}_{ij}$ = constant), and if the deviatoric strain intensity (a form of the second invariant of deviatoric strain) is taken as

$$J_2^e = (0.5 e_{ij}\, e_{ij})^{1/2} \quad (9)$$

then it follows that

$$\dot{e}_I = \frac{e_{ij}\, \dot{\varepsilon}_{ij}}{2J_2^e} \quad (10)$$

Hence, the last term of Eq. 8 can be expressed as

$$[T]\dot{e}_I = \begin{bmatrix} E_3e_{11} & E_3e_{22} & E_3e_{33} & E_3e_{12} & E_3e_{23} & E_3e_{31} \\ E_3e_{11} & E_3e_{22} & E_3e_{33} & E_3e_{12} & E_3e_{23} & E_3e_{31} \\ E_3e_{11} & E_3e_{22} & E_3e_{33} & E_3e_{12} & E_3e_{23} & E_3e_{31} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{Bmatrix} \dot{\varepsilon}_{11} \\ \dot{\varepsilon}_{22} \\ \dot{\varepsilon}_{33} \\ \dot{\varepsilon}_{12} \\ \dot{\varepsilon}_{23} \\ \dot{\varepsilon}_{31} \end{Bmatrix} \quad (11)$$

or $[T]\dot{e}_I = [U]\,\{\dot{\varepsilon}\}$ (12)

in which $E_3 = 3KD/2J_2^e$. Hence, Eq. 8 becomes

$$\{\dot{\sigma}\} = [S - U]\,\{\dot{\varepsilon}\} \quad (13)$$

in which $[S - U]$ is the tangent constitutive matrix, made up of a symmetrical part $[S]$ and an unsymmetrical part $[U]$. The matrix $[U]$ is associated entirely with the shear dilation effect.

4. Piecewise Linear Idealization

Because soil properties are complex and vary widely between soils, it is difficult to establish analytical expressions for the variations of K, G and D. For this reason, a procedure which avoids analytical expressions

has been devised. The features of the procedure are as follows. The technique for establishing numerical values is described subsequently.

A state space is defined, with coordinates J_2^e and $I_1^\sigma = 3\sigma_m$ = first invariant of stress. This space is subdivided into "zones" and "regions", as indicated in Fig. 1. Within any region the values of K, G and D are assumed to be constant. This procedure has the following advantages.

(1) The numbers of J-zones and I-zones can be chosen to provide any desired degree of accuracy.

(2) The values of K, G and D from region to region can be chosen to match any observed soil behavior, without the need for an analytical expression.

(3) The assumption of piecewise constant K, G and D corresponds to piecewise linear behavior between discrete events, which leads to a simple computation strategy.

The invariant I_1^σ is an obvious choice as one coordinate of the state space, because of the dependence of the soil properties on hydrostatic stress. The choice of J_2^e as the second coordinate is preferable to the use of J_2^σ for the following reasons.

(1) It simplifies the creation of J-zones for soils exhibiting strength loss, because J_2^e continues to increase whereas J_2^σ decreases.

(2) The J-zone boundaries can be selected as straight lines in J_2^e space, whereas in J_2^σ space this is more difficult.

(3) During analysis, an "event" occurs when the state moves from one region to the next, and involves intersection of the loading path with the zone boundaries. The intersection problem is easier to solve when the zone boundaries are expressed in terms of J_2^e.

The selection of piecewise constant values for K, G and D ensures that the symmetrical part of the tangent constitutive matrix is piecewise constant. However, the unsymmetrical part of this matrix depends on the ratios e_{ij}/J_2^e, which can vary within a region. To retain the computational advantages of piecewise linear behavior, it is assumed that these ratios remain constant, the values used being those at the end of the load increment in which the region is first entered. This assumption is correct only for radial straining, and for nonradial straining it can lead to a further approximation in the tangent constitutive matrix. This assumption is consistent with the radial straining assumption made in establishing Eq. 11. It does not introduce serious problems, because the nonlinear solution strategy used in the computer analysis automatically iterates to compensate for errors, and because the tangent constitutive matrix is used only in the linearization phase of the computation and not in the state determination phase.

5. Determination of K, G, D Values

In order to establish values for K, G and D in each region of the state space, the results of a hydrostatic compression test are needed,

plus the results of triaxial tests for at least two different confining pressures. The hydrostatic test results should be expressed as a curve relating mean stress to mean strain (Fig. 2) and the triaxial test results as curves relating axial strain to (a) deviator stress ($\sigma_{11} - \sigma_{33}$) and (b) volumetric strain ($\varepsilon_V = 3\sigma_m$) (Fig. 3). The following steps should be followed to obtain numerical values for K, G and D.

Step 1: For each triaxial test curve, draw the corresponding load path in the state space, as shown in Fig. 4a. Select J-zone boundaries such that these load paths are approximated by straight lines as shown, with the desired degree of accuracy. Straight J-zone boundaries are convenient, but not essential to the theory.

Step 2: Transfer the end points of the linear segments from Fig. 4a to the triaxial test curves in Fig. 3. Hence approximate these curves by straight segments as shown. If the approximation is not sufficiently close, modify the zone boundaries in Fig. 4a and repeat.

Step 3: For each linear segment of each curve in the triaxial test results, calculate the tangent shear modulus, G. It can be shown that G is given by

$$G = \frac{\dfrac{\Delta(\sigma_{11} - \sigma_{33})}{\Delta\varepsilon_{11}}}{3(1 - \dfrac{\Delta\varepsilon_m}{\Delta\varepsilon_{11}})} \tag{14}$$

in which $\Delta(\sigma_{11} - \sigma_{33})$, $\Delta\varepsilon_{11}$ and $\Delta\varepsilon_m$ are the increments of deviator stress, axial strain and mean strain, respectively, over a linear segment. At the center of each segment calculate I_1^σ. Hence, for each J-zone plot points and draw a line showing the variation of G with I_1^σ, as indicated in Fig. 4b.

Step 4: From the hydrostatic test results (Fig. 2) construct a curve showing the variation of the tangent bulk modulus, K, with I_1^σ (= $3\sigma_m$), as indicated in Fig. 4c.

Step 5: Considering both the G vs. I_1^σ and K vs. I_1^σ curves, select I-zones with constant G and K values, such that no large or erratic variations occur from zone to zone. Also draw the I-zone boundaries in the $J_2^S - I_1^\sigma$ state space. The I-zones are shown in Figs. 4a, b and c.

Step 6: Consider the paths of the triaxial tests in Fig. 4a. The paths cross the J-zone boundaries and may also cross I-zone boundaries. Represent each path by a series of straight segments, each segment ending where the path crosses a J-zone or I-zone boundary. From Eqs. 1, 2 and 3 it follows that

$$D = \frac{1}{\Delta e_I} (\Delta\varepsilon_m - \frac{\Delta\sigma_m}{3K}) \tag{15}$$

For each linear segment of each triaxial test path, the quantities Δe_I, $\Delta\varepsilon_m$, $\Delta\sigma_m$ and K can be calculated, and hence D can be found. Plot points

showing the variation of D with I_1^σ for each J-zone, and draw lines through these points as indicated in Fig. 4d. Hence, obtain a constant value of D for each region. If the variations of D from region to region are large or erratic, it may be necessary to revise the I-zones and/or J-zones to produce a smoother variation.

6. Unloading

The type of unloading behavior observed in a hydrostatic test is shown in Fig. 2. For analysis, hydrostatic unloading is assumed in any load step if

(a) $\Delta\varepsilon_m^h < 0$; or

(b) $\Delta\varepsilon_m^h \geq 0$ and $\varepsilon_m^h < \max(\varepsilon_m^h)$

in which $\max(\varepsilon_m^h)$ = largest value of the hydrostatic component of mean strain reached previously in the analysis. During unloading the bulk modulus is assumed to be constant within each I-zone, but to be larger than the corresponding loading modulus by a factor f. That is, in any I-zone

$$K_u = f\, K_\ell \tag{16}$$

in which K_ℓ = loading modulus and K_u = unloading modulus.

It may be noted that in the determination of D from the triaxial test paths (Fig. 4a), the mean stress may decrease in some parts of the paths. In this situation, the value K_u has been used in Eq. 14 rather than K_ℓ.

The procedure for considering cyclic distortional straining is still being refined at the time of writing, and will not be considered in detail herein. In essence, unloading occurs if J_2^e decreases. The K and G values are then set to the values for the first J-zone, and D is assumed to be zero. The procedure being developed is actually more complex, the J-zone boundaries being regarded as loading surfaces in a combined stress-strain space, with unloading if the deviatoric strain increment has a negative component normal to the loading surface. The loading surfaces are also assumed to move as the deviatoric strains increase, so that a type of kinematic hardening is simulated. The assumption of a zero value for D is not correct, because actual soils continue to exhibit shear dilation during cyclic straining. This aspect of behavior is complex because sands which expand under monotonic straining may compress under cyclic loading. It is hoped to develop extensions of the procedure to account for this effect.

7. Computer Program

The preceding procedure has been programmed for plane stress, plane strain and axisymmetric solid elements for the general purpose nonlinear analysis code ANSR [7,8,9]. For the linearization phase, an option has been provided to use either the unsymmetric tangent constitutive matrix or only the symmetric part of this matrix. It has been found that use of only the symmetric part leads to substantially slower convegence in the iterative solution. For the state determination phase the calculations

are based on Eq. 7, and hence avoid the radial straining approximation made for the linearization phase. Details of the computational procedure will be presented elsewhere [10].

8. Example

Hydrostatic and triaxial test results for a Sacramento River Sand with an initial void ratio of 0.61 have been reported by Lee [11] and Lee and Seed [12]. The curves shown in Figs. 2, 3 and 4 were taken from these test results. To avoid cluttering the diagrams, only three triaxial test curves and a small number of regions are shown in Figs. 3 and 4, and the experimental points are not shown. For calculating K, G and D, five available triaxial test curves were used, and the state space was divided into six J-zones and fourteen I-zones. It should be noted that the example covers particularly wide ranges of deviatoric strain and mean stress, and that narrower ranges are likely to be sufficient for practical problems.

Analyses have been carried out to simulate hydrostatic, triaxial, uniaxial strain and "simple" shear tests, in order to confirm that the numerically discretized model correctly reproduces the experimentally observed behavior.

The hydrostatic test results were reproduced very closely, as would be expected, and are not shown. The experimental and computed triaxial test results are shown in Fig. 5. The agreement is believed to be very satisfactory considering the complexity of the behavior. The computed results for uniaxial strain are shown in Fig. 6. No experimental test results were available for this case, but the computed behavior agrees qualitatively with reported tests on other sands [13]. The computed results for simple shear tests are shown in Fig. 7. Again, no experimental results were available, but the results are qualitatively correct.

9. Conclusion

The modelling described in this paper has the advantage that it is entirely numerical, and does not rely on analytical expressions for the soil properties. As a result it can be adapted to essentially any type of observed soil behavior. The procedure also has the advantage that shear dilation effects are considered, whereas these effects are usually ignored in variable modulus formulations. A significant amount of computation is required to calculate numerical values for the tangent moduli, but this effort is small compared with the total effort required to obtain the soil properties. The procedure has been shown to reproduce the test results on which it is based, and hence it is probable that actual soil configurations can be analyzed accurately.

A fundamental disadvantage of the procedure is that it is based on observed behavior, and does not account directly for the fundamental mechanism involved. This disadvantage is shared with virtually all other available soil models. Other limitations are that time-dependent effects have not been considered, and that cyclic loading effects are not correctly modelled, especially with regard to compaction of the soil. It is hoped that these limitations can be removed as the procedure is developed further.

References

1. Desai, C. S., "Overview, Trends and Projections, Theory and Application of the Finite Element Method in Geotechnical Engineering", State-of-the-Art, Proc. of the Symposium held at Vicksburg, Mississippi, edited by C. S. Desai, Vol. 1, pp. 3-90, May 1972.

2. Nelson, I. and Baron, M., "Investigation of Ground Shock Effects in Nonlinear Hysteretic Media," Report 1, Development of Mathematical Material Models, U.S. Army Engineer Waterways Experimental Station, DACA 39-67-C-0048, Contract Report S-68-1, March 1968.

3. Nelson, I. and Baron, M., "Application of Variable Moduli Models to Soil Behavior," Int. J. Solids and Structures, Vol. 7, pp. 399-417, 1971.

4. Lomize, G. M., and Kryzhanovskii, A. L., "Soil Strength," Gidrotekhnicheskoe Stroitel'stvo, No. 3, March 1967.

5. Vyalov, S. S., "Problems of the Theory of Deformability of Cohesive Soils," Fundamenty i Mekhanika Gruntor, No. 3, May-June 1966.

6. Bazant, Z, Bhat, P., and Shieh, C., "Endochronic Theory for Inelastic and Failure Analysis of Concrete Structures," Structural Engineering Report, No. 1976-12/259, Department of Civil Engineering,Northwestern University, Dec. 1976.

7. Mondkar, D. P. and Powell, G. H., "Static and Dynamic Analysis of Nonlinear Structures," Report No. EERC 75-10, Earthquake Engineering Research Center, University of California, Berkeley, 1975.

8. Mondkar, D. P. and Powell, G. H., "ANSR-I General Purpose Program for Analysis of Nonlinear Structural Response," Report No. EERC 75-37, Earthquake Engineering Research Center, University of California, Berkeley, 1975.

9. Mondkar, D. P. and Powell, G. H., "Finite Element Analysis of Non-linear Static and Dynamic Response," Int. J. for Numerical Methods in Engineering, Vol. II, pp. 499-520, 1977.

10. Al-Shawaf, T. D., "Variable Modulus Model in Non-linear Finite Element Analysis," Ph.D. Dissertation, University of California, Berkeley, (In preparation).

11. Lee, K. L., "Triaxial Compression Strength of Saturated Sand Under Seismic Loading Conditions," Ph.D. Dissertation, University of California, Berkeley, 1965.

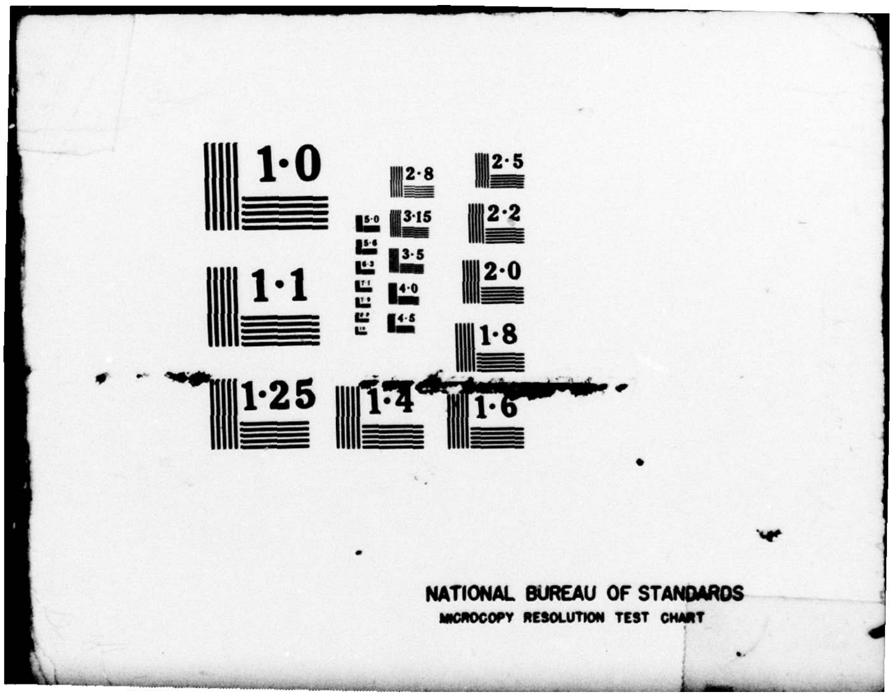
12. Lee, K. L. and Seed, H. B., "Drained Strength Characteristics of Sands," J. of Soil Mechanics and Found. Div., ASCE, Vol. 93, No. SM6, pp. 117-141, Nov. 1967.

13. Lamb, T. W. and Whitman, R. V., "Soil Mechanics," John Wiley and Sons, Inc., 1969.

AD-A068 222 CALIFORNIA UNIV BERKELEY GEOTECHNICAL ENGINEERING F/G 8/13
EVALUATION OF THREE CONSTITUTIVE MODELS FOR SOILS.(U)
MAR 79 P C LUCIA, J M DUNCAN DACW39-78-M-2077
UNCLASSIFIED UCB/GT/78-05 WES-MPGL-79-3 NL

2 OF 2
ADA
068222

END
DATE
FILMED
6-79
DDC


1·0
2·8
2·5
3·15
2·2
3·5
2·0
4·0
1·1
1·8
1·25
1·4
1·6
NATIONAL BUREAU OF STANDARDS
MICROCOPY RESOLUTION TEST CHART

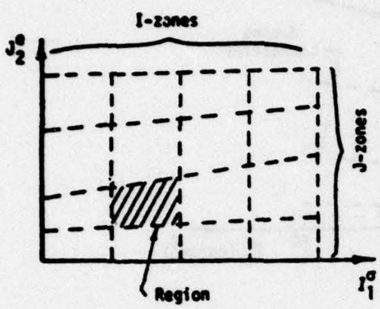


FIG. 1 SUBDIVISION OF STATE SPACE

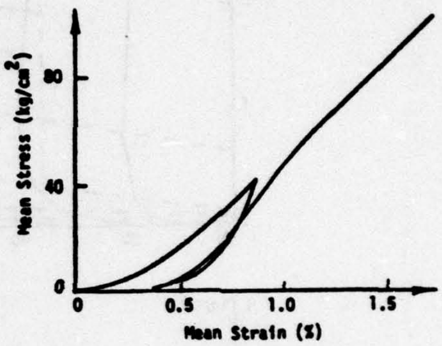
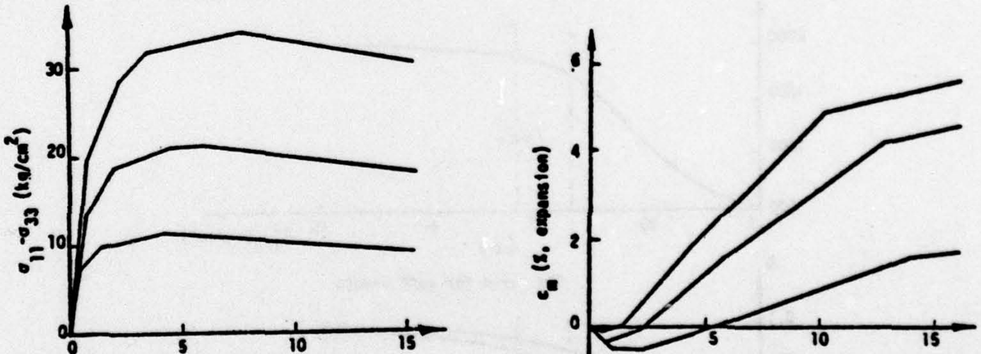


FIG. 2 TYPICAL HYDROSTATIC TEST RESULTS



(a) (b)

FIG. 3 TYPICAL TRIAXIAL TEST RESULTS

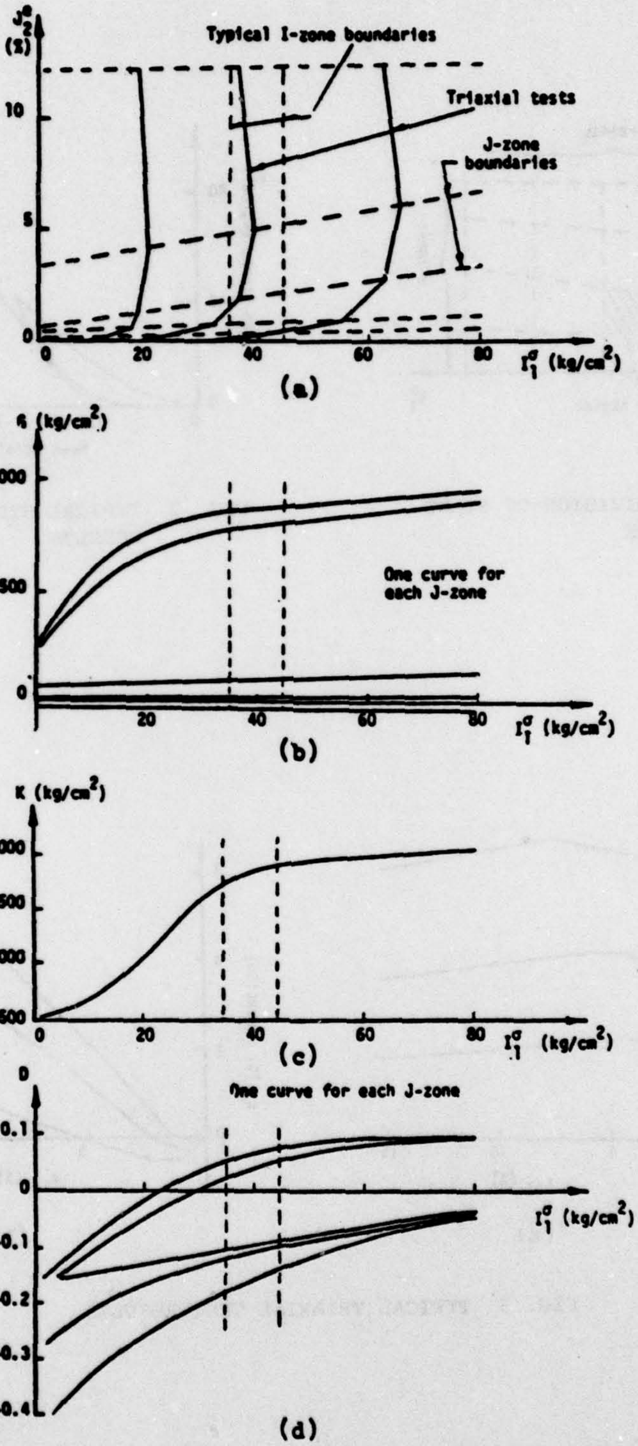


FIG. 4 DETERMINATION OF K, G, D VALUES

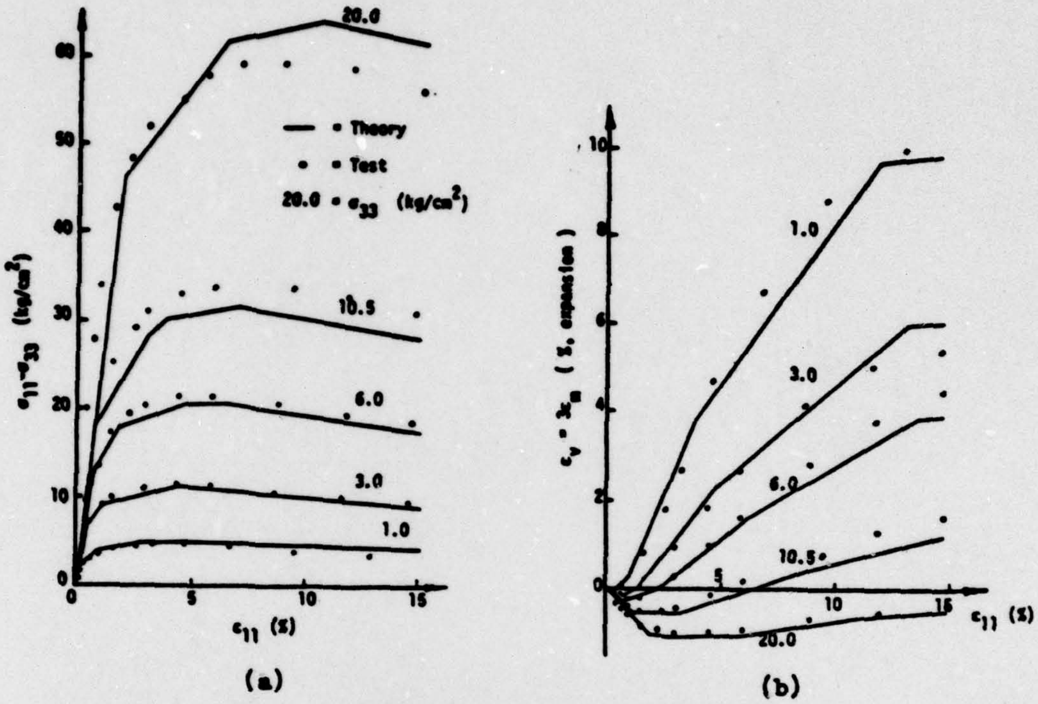


FIG. 5 COMPUTED TRIAXIAL TEST RESULTS

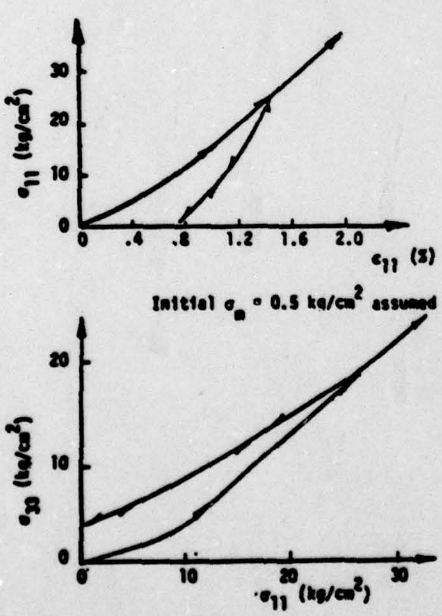


FIG. 6 COMPUTED UNIAXIAL STRAIN TEST RESULTS

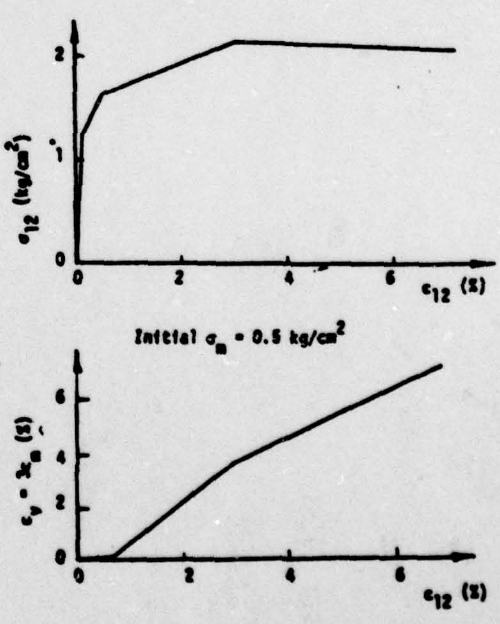


FIG. 7 COMPUTED SIMPLE SHEAR TEST RESULTS

In accordance with letter from DAEN-RDC, DAEN-ASI dated 22 July 1977, Subject: Facsimile Catalog Cards for Laboratory Technical Publications, a facsimile catalog card in Library of Congress MARC format is reproduced below.

Lucia, Patrick C
Evaluation of three constitutive models for soils / by Patrick C. Lucia, James M. Duncan, College of Engineering, University of California, Berkeley, California. Vicksburg, Miss. : U. S. Waterways Experiment Station ; Springfield, Va. : available from National Technical Information Service, 1979.
iii, 81, 11 p. : ill. ; 27 cm. (Miscellaneous paper - U. S. Army Engineer Waterways Experiment Station ; GL-79-3)
Prepared for Office, Chief of Engineers, U. S. Army, Washington, D. C., under Contract No. DACW39-78-M-2077, Work Unit CWIS 31173, Task 25c.
Includes bibliographies.
1. Constitutive models. 2. Finite element method. 3. Dilatancy (Soils). 4. Soil deformation. 5. Stress-strain relations (Soils). 6. Mathematical models. I. Duncan, James Michael, joint author. II. California. University. College of Engineering. III. United States. Army. Corps of Engineers. IV. Series: United States. Waterways Experiment Station, Vicksburg, Miss. Miscellaneous paper ; GL-79-3.
TA7.W34m no.GL-79-3